第四次作业

2020年12月25日 星期五　　下午10:19

4、对于 $j = 1, 2, \cdots, n+1$，令 $f(x) = x^j$

考虑 $f(x)$ 的 n 阶 Lagrange 插值.

$$L_n(x) = \sum_{i=0}^{n} l_i(x) f(x_i) = \sum_{i=0}^{n} l_i(x)\, x_i^j$$

余项为：$R_n(x) = f(x) - L_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi(x))\, \omega_{n+1}(x)$

则 $j \le n$ 时 $R_n = 0$，　$j = n+1$ 时 $R_n = \omega_{n+1}(x)$

令 $x=0$ $\Rightarrow$ $\sum_{i=0}^{n} l_i(0)\, x_i^j = \begin{cases} 0, & j = 1, 2, \cdots, n \\ (-1)^n x_0 \cdots x_n, & j = n+1. \end{cases}$

8. (1) $N_1(x) = 2(\sqrt{e} - 1)x + 1$

$N_1(0.25) \approx 1.324$

(2) $N_2(x) = \frac{(e-1)^2}{2} x(x-1) + (e-1)x + 1$

$\Rightarrow N_2(0.25) \approx 1.153$　　$N_2(0.75) \approx 2.012$

16. 1° $0 \le x \le 1$　$S(x) = -x^3 + 2x + 1$

$S'(x) = -3x^2 + 1$

$S''(x) = -6x$

2° $1 \le x \le 2$　$S(x) = d(x-1)^3 + c(x-1)^2 + b(x-1) + 2$

$S'(x) = 3dx^2 + 2(c - 3d)x + (3d - 2c + b)$

$S''(x) = 6dx + 2(c - 3d)$

则 考虑在 $x=1$ 处 保证 $S(x)$ 二阶可导，

且 $S''(0) = S''(2) = 0$

$\Rightarrow$　$b = -1$，$c = -3$，$d = 1$