球对称metric

```
restart
with(tensor) :
coord := [t, r, θ, φ]
```

$$[t,\ r,\ \theta,\ \phi]$$

```
g_compts := array(symmetric, sparse, 1..4, 1..4)
```

$$\mathit{array}(\mathit{symmetric},\ \mathit{sparse},\ 1..4,\ 1..4,\ [\,])$$

$g\_compts_{1,\,1} := \exp(2 \cdot A(t,\,r))$: $g\_compts_{2,\,2} := -\exp(2 \cdot B(t,\,r))$:

$g\_compts_{3,\,3} := -r^2$: $g\_compts_{4,\,4} := -r^2 \sin(\theta)^2$: $g := \mathit{create}([-1,\,-1],\ \mathit{eval}(g\_compts))$

$$\mathit{table}\left(\left[\mathit{index\_char} = [-1,\,-1],\ \mathit{compts} = \begin{bmatrix} e^{2\,A(t,\,r)} & 0 & 0 & 0 \\ 0 & -e^{2\,B(t,\,r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg')
D1g := d1metric(g, coord) : D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1,\,4,\,1,\,4] = -\frac{\left(\frac{\partial}{\partial r} A(t,\,r)\right) e^{2\,A(t,\,r)}\, r \sin(\theta)^2}{e^{2\,B(t,\,r)}},\ [2,\,4,\,2,\,4] = -\left(\frac{\partial}{\partial r} B(t,\,r)\right) r \sin(\theta)^2,\ [3,\,4,\,3,\,4] = \frac{r^2 \left(\cos(\theta)^2 e^{2\,B(t,\,r)} + 1 - \cos(\theta)^2 - e^{2\,B(t,\,r)}\right)}{e^{2\,B(t,\,r)}},\right.$$

$$[2,\,3,\,2,\,3] = -\left(\frac{\partial}{\partial r} B(t,\,r)\right) r,\ [1,\,3,\,1,\,3] = -\frac{\left(\frac{\partial}{\partial r} A(t,\,r)\right) e^{2\,A(t,\,r)}\, r}{e^{2\,B(t,\,r)}},\ [1,\,2,\,1,\,2] = \left(\frac{\partial}{\partial r} B(t,\,r)\right)\left(\frac{\partial}{\partial r} A(t,\,r)\right) e^{2\,A(t,\,r)} + \left(\frac{\partial}{\partial t} B(t,\,r)\right)^2 e^{2\,B(t,\,r)}$$

$$- \left(\frac{\partial}{\partial t} B(t,\,r)\right)\left(\frac{\partial}{\partial t} A(t,\,r)\right) e^{2\,B(t,\,r)} - \left(\frac{\partial}{\partial r} A(t,\,r)\right)^2 e^{2\,A(t,\,r)}$$

$$\left. + \left(\frac{\partial^2}{\partial t^2} B(t,\,r)\right) e^{2\,B(t,\,r)} - \left(\frac{\partial^2}{\partial r^2} A(t,\,r)\right) e^{2\,A(t,\,r)},\ [1,\,3,\,2,\,3] = -\left(\frac{\partial}{\partial t} B(t,\,r)\right) r,\ [1,\,4,\,2,\,4] = -\left(\frac{\partial}{\partial t} B(t,\,r)\right) r \sin(\theta)^2\right]$$

```
RICCI := Ricci(ginv, RMN)
```

$$
\mathit{table}\Bigg(\Bigg[\mathit{index\_char} = [-1, -1],\ \mathit{compts} = \Bigg[\Bigg[\frac{1}{r\, \mathrm{e}^{2\, B(t,\, r)}}\Bigg(\left(\frac{\partial}{\partial r} B(t, r)\right)\left(\frac{\partial}{\partial r} A(t, r)\right) \mathrm{e}^{2\, A(t,\, r)}\, r + \left(\frac{\partial}{\partial t} B(t, r)\right)^2 \mathrm{e}^{2\, B(t,\, r)}\, r - \left(\frac{\partial}{\partial t} B(t, r)\right)\left(\frac{\partial}{\partial t} A(t, r)\right) \mathrm{e}^{2\, B(t,\, r)}\, r - \left(\frac{\partial}{\partial r} A(t, r)\right)^2 \mathrm{e}^{2\, A(t,\, r)}\, r + \left(\frac{\partial^2}{\partial t^2} B(t, r)\right) \mathrm{e}^{2\, B(t,\, r)}\, r - \left(\frac{\partial^2}{\partial r^2} A(t, r)\right) \mathrm{e}^{2\, A(t,\, r)}\, r - 2\left(\frac{\partial}{\partial r} A(t, r)\right) \mathrm{e}^{2\, A(t,\, r)}\Bigg),\ -\frac{2\left(\frac{\partial}{\partial t} B(t, r)\right)}{r},\ 0,\ 0\Bigg],
$$

$$
\Bigg[-\frac{2\left(\frac{\partial}{\partial t} B(t, r)\right)}{r},\ -\frac{1}{\mathrm{e}^{2\, A(t,\, r)}\, r}\Bigg(\left(\frac{\partial}{\partial r} B(t, r)\right)\left(\frac{\partial}{\partial r} A(t, r)\right) \mathrm{e}^{2\, A(t,\, r)}\, r + \left(\frac{\partial}{\partial t} B(t, r)\right)^2 \mathrm{e}^{2\, B(t,\, r)}\, r - \left(\frac{\partial}{\partial t} B(t, r)\right)\left(\frac{\partial}{\partial t} A(t, r)\right) \mathrm{e}^{2\, B(t,\, r)}\, r - \left(\frac{\partial}{\partial r} A(t, r)\right)^2 \mathrm{e}^{2\, A(t,\, r)}\, r + \left(\frac{\partial^2}{\partial t^2} B(t, r)\right) \mathrm{e}^{2\, B(t,\, r)}\, r - \left(\frac{\partial^2}{\partial r^2} A(t, r)\right) \mathrm{e}^{2\, A(t,\, r)}\, r + 2\left(\frac{\partial}{\partial r} B(t, r)\right) \mathrm{e}^{2\, A(t,\, r)}\Bigg),\ 0,\ 0\Bigg],
$$

$$
\Bigg[0,\ 0,\ -\frac{\left(\frac{\partial}{\partial r} B(t, r)\right) r - \left(\frac{\partial}{\partial r} A(t, r)\right) r + \mathrm{e}^{2\, B(t,\, r)} - 1}{\mathrm{e}^{2\, B(t,\, r)}},\ 0\Bigg],
$$

$$
\Bigg[0,\ 0,\ 0,\ \frac{1}{\mathrm{e}^{2\, B(t,\, r)}}\Bigg(\cos(\theta)^2\left(\frac{\partial}{\partial r} B(t, r)\right) r - \cos(\theta)^2\left(\frac{\partial}{\partial r} A(t, r)\right) r + \cos(\theta)^2\, \mathrm{e}^{2\, B(t,\, r)} - \cos(\theta)^2 - \left(\frac{\partial}{\partial r} B(t, r)\right) r + \left(\frac{\partial}{\partial r} A(t, r)\right) r - \mathrm{e}^{2\, B(t,\, r)} + 1\Bigg)\Bigg]\Bigg]\Bigg]\Bigg)
$$

$RS := \mathit{Ricciscalar}(\mathit{ginv}, \mathit{RICCI})$

$$table\left(\left[index\_char=[\,],\ compts=\frac{1}{e^{2\,A(t,\,r)}\,e^{2\,B(t,\,r)}\,r^2}\left(2\,\left(\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\,\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\,e^{2\,A(t,\,r)}\,r^2+\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)^2\,e^{2\,B(t,\,r)}\,r^2-\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)\,\left(\frac{\partial}{\partial t}\,A(t,\,r)\right)\,e^{2\,B(t,\,r)}\,r^2-\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)^2\,e^{2\,A(t,\,r)}\,r^2+\left(\frac{\partial^2}{\partial t^2}\,B(t,\,r)\right)\,e^{2\,B(t,\,r)}\,r^2-\left(\frac{\partial^2}{\partial r^2}\,A(t,\,r)\right)\,e^{2\,A(t,\,r)}\,r^2+2\,\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\,e^{2\,A(t,\,r)}\,r-2\,\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\,e^{2\,A(t,\,r)}\,r+e^{2\,A(t,\,r)}\,e^{2\,B(t,\,r)}-e^{2\,A(t,\,r)}\right)\right)\right]\right)$$

$Estn := Einstein(g,\ RICCI,\ RS)$

Schildwarzschild

$restart$

$with(tensor):$

$coord := [t, r, \theta, \phi]$

$$[t, r, \theta, \phi]$$

$g\_compts := array(symmetric, sparse, 1..4, 1..4)$

$$array(symmetric, sparse, 1..4, 1..4, [\,])$$

$g\_compts_{1,1} := 1 - \frac{2\,M}{r} : \; g\_compts_{2,2} := -\left(1 - \frac{2\,M}{r}\right)^{-1} :$

$g\_compts_{3,3} := -r^2 : \; g\_compts_{4,4} := -r^2 \sin(\theta)^2 : \; g := create([-1, -1], eval(g\_compts))$

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} 1 - \frac{2\,M}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1 - \frac{2\,M}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}\right]\right)$$

$ginv := invert(g, 'detg'):$

$D1g := d1metric(g, coord): \; D2g := d2metric(D1g, coord):$

$Cf1 := Christoffel1(D1g):$

$RMN := Riemann(ginv, D2g, Cf1):$

$RMNc := get\_compts(RMN):$

```
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[2, 4, 2, 4] = -\frac{M \sin(\theta)^2}{-r + 2\,M}, [2, 3, 2, 3] = -\frac{M}{-r + 2\,M}, [1, 3, 1, 3] = \frac{(-r + 2\,M)\,M}{r^2}, [1, 2, 1, 2] = \frac{2\,M}{r^3}, [1, 4, 1, 4] = \frac{(-r + 2\,M)\,M \sin(\theta)^2}{r^2}, [3, 4, 3, 4] = -2\,r\,M \sin(\theta)^2\right]$$

$RICCI := Ricci(ginv, RMN)$

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right]\right)$$

$RS := Ricciscalar(ginv, RICCI)$

$$table([index\_char = [\,], compts = 0])$$

$Estn := Einstein(g, RICCI, RS)$

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\right]\right)$$

$Cf2 := Christoffel2(ginv,\ Cf1)$

$$table\Big(\Big[index\_char=[1,\ -1,\ -1],\ compts=ARRAY\Big(cf2,\ [1..4,\ 1..4,\ 1..4],\ \Big[(1,\ 1,\ 1)=0,\ (1,\ 1,\ 2)=-\frac{M}{r\ (-r+2\ M)},\ (1,\ 1,\ 3)=0,\ (1,\ 1,\ 4)=0,\ (1,\ 2,\ 1)=-\frac{M}{r\ (-r+2\ M)},\ (1,\ 2,\ 2)=0,\ (1,\ 2,\ 3)=0,\ (1,\ 2,\ 4)=0,\ (1,\ 3,\ 1)=0,\ (1,\ 3,\ 2)=0,\ (1,\ 3,\ 3)=0,\ (1,\ 3,\ 4)=0,\ (1,\ 4,\ 1)=0,\ (1,\ 4,\ 2)=0,\ (1,\ 4,\ 3)=0,\ (1,\ 4,\ 4)=0,\ (2,\ 1,\ 1)=-\frac{(-r+2\ M)\ M}{r^3},\ (2,\ 1,\ 2)=0,\ (2,\ 1,\ 3)=0,\ (2,\ 1,\ 4)=0,\ (2,\ 2,\ 1)=0,\ (2,\ 2,\ 2)=\frac{M}{r\ (-r+2\ M)},\ (2,\ 2,\ 3)=0,\ (2,\ 2,\ 4)=0,\ (2,\ 3,\ 1)=0,\ (2,\ 3,\ 2)=0,\ (2,\ 3,\ 3)=-r+2\ M,\ (2,\ 3,\ 4)=0,\ (2,\ 4,\ 1)=0,\ (2,\ 4,\ 2)=0,\ (2,\ 4,\ 3)=0,\ (2,\ 4,\ 4)=(-r+2\ M)\ \sin(\theta)^2,\ (3,\ 1,\ 1)=0,\ (3,\ 1,\ 2)=0,\ (3,\ 1,\ 3)=0,\ (3,\ 1,\ 4)=0,\ (3,\ 2,\ 1)=0,\ (3,\ 2,\ 2)=0,\ (3,\ 2,\ 3)=\frac{1}{r},\ (3,\ 2,\ 4)=0,\ (3,\ 3,\ 1)=0,\ (3,\ 3,\ 2)=\frac{1}{r},\ (3,\ 3,\ 3)=0,\ (3,\ 3,\ 4)=0,\ (3,\ 4,\ 1)=0,\ (3,\ 4,\ 2)=0,\ (3,\ 4,\ 3)=0,\ (3,\ 4,\ 4)=-\sin(\theta)\ \cos(\theta),\ (4,\ 1,\ 1)=0,\ (4,\ 1,\ 2)=0,\ (4,\ 1,\ 3)=0,\ (4,\ 1,\ 4)=0,\ (4,\ 2,\ 1)=0,\ (4,\ 2,\ 2)=0,\ (4,\ 2,\ 3)=0,\ (4,\ 2,\ 4)=\frac{1}{r},\ (4,\ 3,\ 1)=0,\ (4,\ 3,\ 2)=0,\ (4,\ 3,\ 3)=0,\ (4,\ 3,\ 4)=\frac{\cos(\theta)}{\sin(\theta)},\ (4,\ 4,\ 1)=0,\ (4,\ 4,\ 2)=\frac{1}{r},\ (4,\ 4,\ 3)=\frac{\cos(\theta)}{\sin(\theta)},\ (4,\ 4,\ 4)=0\Big]\Big)\Big]\Big)$$

$$
\mathit{table}\Bigg(\Bigg[\mathit{index\_char}=[-1,\,-1],\ \mathit{compts}=\Bigg[\Bigg[-\frac{\mathrm{e}^{2\,A(t,\,r)}\left(2\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)r+\mathrm{e}^{2\,B(t,\,r)}-1\right)}{\mathrm{e}^{2\,B(t,\,r)}\,r^{2}},\ -\frac{2\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)}{r},\ 0,\ 0\Bigg],
$$

$$
\Bigg[-\frac{2\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)}{r},\ -\frac{2\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)r-\mathrm{e}^{2\,B(t,\,r)}+1}{r^{2}},\ 0,\ 0\Bigg],
$$

$$
\Bigg[0,\ 0,\ \frac{1}{\mathrm{e}^{2\,A(t,\,r)}\,\mathrm{e}^{2\,B(t,\,r)}}\Bigg(r\Bigg(\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\,r
+\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)^{2}\mathrm{e}^{2\,B(t,\,r)}\,r-\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)\left(\frac{\partial}{\partial t}\,A(t,\,r)\right)\mathrm{e}^{2\,B(t,\,r)}\,r
-\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)^{2}\mathrm{e}^{2\,A(t,\,r)}\,r+\left(\frac{\partial^{2}}{\partial t^{2}}\,B(t,\,r)\right)\mathrm{e}^{2\,B(t,\,r)}\,r-\left(\frac{\partial^{2}}{\partial r^{2}}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\,r+\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}-\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\Bigg)\Bigg),\ 0\Bigg],
$$

$$
\Bigg[0,\ 0,\ 0,\ -\frac{1}{\mathrm{e}^{2\,A(t,\,r)}\,\mathrm{e}^{2\,B(t,\,r)}}\Bigg(r\Bigg(\cos(\theta)^{2}\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\,r+\cos(\theta)^{2}\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)^{2}\mathrm{e}^{2\,B(t,\,r)}\,r-\cos(\theta)^{2}\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)\left(\frac{\partial}{\partial t}\,A(t,\,r)\right)\mathrm{e}^{2\,B(t,\,r)}\,r-\cos(\theta)^{2}\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)^{2}\mathrm{e}^{2\,A(t,\,r)}\,r
+\left(\frac{\partial^{2}}{\partial t^{2}}\,B(t,\,r)\right)\cos(\theta)^{2}\,\mathrm{e}^{2\,B(t,\,r)}\,r-\left(\frac{\partial^{2}}{\partial r^{2}}\,A(t,\,r)\right)\cos(\theta)^{2}\,\mathrm{e}^{2\,A(t,\,r)}\,r
+\cos(\theta)^{2}\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}-\cos(\theta)^{2}\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}
-\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\,r-\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)^{2}\mathrm{e}^{2\,B(t,\,r)}\,r
+\left(\frac{\partial}{\partial t}\,B(t,\,r)\right)\left(\frac{\partial}{\partial t}\,A(t,\,r)\right)\mathrm{e}^{2\,B(t,\,r)}\,r+\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)^{2}\mathrm{e}^{2\,A(t,\,r)}\,r
-\left(\frac{\partial^{2}}{\partial t^{2}}\,B(t,\,r)\right)\mathrm{e}^{2\,B(t,\,r)}\,r+\left(\frac{\partial^{2}}{\partial r^{2}}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\,r-\left(\frac{\partial}{\partial r}\,B(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}+\left(\frac{\partial}{\partial r}\,A(t,\,r)\right)\mathrm{e}^{2\,A(t,\,r)}\Bigg)\Bigg)\Bigg]\Bigg]\Bigg]\Bigg)
$$

$Cf2 := Christoffel2(ginv,\ Cf1)$

$$table\left(\left[index\_char = [1, -1, -1], compts = ARRAY\left(cf2, [1..4, 1..4, 1..4], \left[(1, 1, 1) = \frac{\partial}{\partial t} A(t, r), (1, 1, 2) = \frac{\partial}{\partial r} A(t, r), (1, 1, 3) = 0, (1, 1, 4) = 0, (1, 2, 1) = \frac{\partial}{\partial r} A(t, r), (1, 2, 2) = \frac{\left(\frac{\partial}{\partial t} B(t, r)\right) e^{2 B(t, r)}}{e^{2 A(t, r)}}, (1, 2, 3) = 0, (1, 2, 4) = 0, (1, 3, 1) = 0, (1, 3, 2) = 0, (1, 3, 3) = 0, (1, 3, 4) = 0, (1, 4, 1) = 0, (1, 4, 2) = 0, (1, 4, 3) = 0, (1, 4, 4) = 0, (2, 1, 1) = \frac{\left(\frac{\partial}{\partial r} A(t, r)\right) e^{2 A(t, r)}}{e^{2 B(t, r)}}, (2, 1, 2) = \frac{\partial}{\partial t} B(t, r), (2, 1, 3) = 0, (2, 1, 4) = 0, (2, 2, 1) = \frac{\partial}{\partial t} B(t, r), (2, 2, 2) = \frac{\partial}{\partial r} B(t, r), (2, 2, 3) = 0, (2, 2, 4) = 0, (2, 3, 1) = 0, (2, 3, 2) = 0, (2, 3, 3) = -\frac{r}{e^{2 B(t, r)}}, (2, 3, 4) = 0, (2, 4, 1) = 0, (2, 4, 2) = 0, (2, 4, 3) = 0, (2, 4, 4) = -\frac{r \sin(\theta)^2}{e^{2 B(t, r)}}, (3, 1, 1) = 0, (3, 1, 2) = 0, (3, 1, 3) = 0, (3, 1, 4) = 0, (3, 2, 1) = 0, (3, 2, 2) = 0, (3, 2, 3) = \frac{1}{r}, (3, 2, 4) = 0, (3, 3, 1) = 0, (3, 3, 2) = \frac{1}{r}, (3, 3, 3) = 0, (3, 3, 4) = 0, (3, 4, 1) = 0, (3, 4, 2) = 0, (3, 4, 3) = 0, (3, 4, 4) = -\sin(\theta) \cos(\theta), (4, 1, 1) = 0, (4, 1, 2) = 0, (4, 1, 3) = 0, (4, 1, 4) = 0, (4, 2, 1) = 0, (4, 2, 2) = 0, (4, 2, 3) = 0, (4, 2, 4) = \frac{1}{r}, (4, 3, 1) = 0, (4, 3, 2) = 0, (4, 3, 3) = 0, (4, 3, 4) = \frac{\cos(\theta)}{\sin(\theta)}, (4, 4, 1) = 0, (4, 4, 2) = \frac{1}{r}, (4, 4, 3) = \frac{\cos(\theta)}{\sin(\theta)}, (4, 4, 4) = 0\right]\right)\right]\right)$$

球对称真空metric

```
restart
with(tensor) :
coord := [t, r, θ, ϕ]
```

$$[t, r, \theta, \phi]$$

```
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```

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$g\_compts_{1,1} := \exp(2\cdot A(r))$: $g\_compts_{2,2} := -\exp(2\cdot B(r))$:

$g\_compts_{3,3} := -r^2$: $g\_compts_{4,4} := -r^2 \sin(\theta)^2$: $g := create([-1, -1], eval(g\_compts))$

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} e^{2\ A(r)} & 0 & 0 & 0 \\ 0 & -e^{2\ B(r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg')
```

$$table\left(\left[index\_char = [1, 1], compts = \begin{bmatrix} \frac{1}{e^{2\ A(r)}} & 0 & 0 & 0 \\ 0 & -\frac{1}{e^{2\ B(r)}} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2 \sin(\theta)^2} \end{bmatrix}\right]\right)$$

```
D1g := d1metric(g, coord) : D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1, 2, 1, 2] = -\left(\frac{d^2}{dr^2} A(r)\right) e^{2 A(r)} - \left(\frac{d}{dr} A(r)\right)^2 e^{2 A(r)} + \left(\frac{d}{dr} A(r)\right) e^{2 A(r)} \left(\frac{d}{dr} B(r)\right), [1, 4, 1, 4] = -\frac{\left(\frac{d}{dr} A(r)\right) e^{2 A(r)} r \sin(\theta)^2}{e^{2 B(r)}},\right.$$

$$[3, 4, 3, 4] = \frac{r^2 \left(\cos(\theta)^2 e^{2 B(r)} + 1 - \cos(\theta)^2 - e^{2 B(r)}\right)}{e^{2 B(r)}}, [1, 3, 1, 3] = -\frac{\left(\frac{d}{dr} A(r)\right) e^{2 A(r)} r}{e^{2 B(r)}}, [2, 4, 2, 4] = -\left(\frac{d}{dr} B(r)\right) r \sin(\theta)^2, [2, 3, 2, 3] =$$

$$\left.-\left(\frac{d}{dr} B(r)\right) r\right]$$

$RICCI := Ricci(ginv, RMN)$

$$table\left(\left[index\_char = [-1, -1], compts = \left[\left[-\frac{1}{r\, e^{2 B(r)}}\left(e^{2 A(r)} \left(-\left(\frac{d}{dr} B(r)\right)\left(\frac{d}{dr} A(r)\right) r + \left(\frac{d}{dr} A(r)\right)^2 r + \left(\frac{d^2}{dr^2} A(r)\right) r + 2\left(\frac{d}{dr} A(r)\right)\right)\right), 0, 0, 0\right],\right.\right.\right.$$

$$\left[0, \frac{-\left(\frac{d}{dr} B(r)\right)\left(\frac{d}{dr} A(r)\right) r + \left(\frac{d}{dr} A(r)\right)^2 r + \left(\frac{d^2}{dr^2} A(r)\right) r - 2\left(\frac{d}{dr} B(r)\right)}{r}, 0, 0\right],$$

$$\left[0, 0, -\frac{\left(\frac{d}{dr} B(r)\right) r - \left(\frac{d}{dr} A(r)\right) r + e^{2 B(r)} - 1}{e^{2 B(r)}}, 0\right],$$

$$\left[0, 0, 0, \frac{1}{e^{2 B(r)}}\left(\cos(\theta)^2 \left(\frac{d}{dr} B(r)\right) r - \cos(\theta)^2 \left(\frac{d}{dr} A(r)\right) r + \cos(\theta)^2 e^{2 B(r)} - \cos(\theta)^2 - \left(\frac{d}{dr} B(r)\right) r + \left(\frac{d}{dr} A(r)\right) r - e^{2 B(r)} + 1\right)\right]\left.\left.\left.\right]\right]\right)$$

$RS := Ricciscalar(ginv, RICCI)$

$$table\left(\left[index\_char=[\,],\ compts=-\frac{1}{e^{2\,B(r)}\,r^2}\left(2\left(-\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)r^2+\left(\frac{d}{dr}A(r)\right)^2 r^2+\left(\frac{d^2}{dr^2}A(r)\right)r^2-2\left(\frac{d}{dr}B(r)\right)r+2\left(\frac{d}{dr}A(r)\right)r-e^{2\,B(r)}+1\right)\right)\right]\right)$$

$Estn := Einstein(g, RICCI, RS)$

$$table\left(\left[index\_char=[-1,-1],\ compts=\left[\left[-\frac{e^{2\,A(r)}\left(2\left(\frac{d}{dr}B(r)\right)r+e^{2\,B(r)}-1\right)}{e^{2\,B(r)}\,r^2},\ 0,\ 0,\ 0\right],\right.\right.\right.$$

$$\left[0,\ -\frac{2\left(\frac{d}{dr}A(r)\right)r-e^{2\,B(r)}+1}{r^2},\ 0,\ 0\right],$$

$$\left[0,\ 0,\ -\frac{1}{e^{2\,B(r)}}\left(r\left(-\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)r+\left(\frac{d}{dr}A(r)\right)^2 r+\left(\frac{d^2}{dr^2}A(r)\right)r-\left(\frac{d}{dr}B(r)\right)+\frac{d}{dr}A(r)\right)\right),\ 0\right],$$

$$\left[0,\ 0,\ 0,\ \frac{1}{e^{2\,B(r)}}\left(r\left(-\cos(\theta)^2\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)r+\cos(\theta)^2\left(\frac{d}{dr}A(r)\right)^2 r+\left(\frac{d^2}{dr^2}A(r)\right)\cos(\theta)^2 r-\cos(\theta)^2\left(\frac{d}{dr}B(r)\right)+\cos(\theta)^2\left(\frac{d}{dr}A(r)\right)+\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)r-\left(\frac{d}{dr}A(r)\right)^2 r-\left(\frac{d^2}{dr^2}A(r)\right)r+\frac{d}{dr}B(r)-\left(\frac{d}{dr}A(r)\right)\right)\right)\right]\left.\left.\left.\right]\right]\right)$$

$Cf2 := Christoffel2(ginv, Cf1)$

$$table\left(\left[index\_char=[1,\ -1,\ -1],\ compts=ARRAY\left(cf2,\ [1..4,\ 1..4,\ 1..4],\ \left[(1,\ 1,\ 1)=0,\ (1,\ 1,\ 2)=\frac{d}{dr}\ A(r),\ (1,\ 1,\ 3)=0,\ (1,\ 1,\ 4)=0,\ (1,\ 2,\ 1)=\frac{d}{dr}\ A(r),\ (1,\ 2,\ 2)=0,\ (1,\ 2,\ 3)=0,\ (1,\ 2,\ 4)=0,\ (1,\ 3,\ 1)=0,\ (1,\ 3,\ 2)=0,\ (1,\ 3,\ 3)=0,\ (1,\ 3,\ 4)=0,\ (1,\ 4,\ 1)=0,\ (1,\ 4,\ 2)=0,\ (1,\ 4,\ 3)=0,\ (1,\ 4,\ 4)=0,\ (2,\ 1,\ 1)=\frac{\left(\frac{d}{dr}\ A(r)\right)\ e^{2\ A(r)}}{e^{2\ B(r)}},\ (2,\ 1,\ 2)=0,\ (2,\ 1,\ 3)=0,\ (2,\ 1,\ 4)=0,\ (2,\ 2,\ 1)=0,\ (2,\ 2,\ 2)=\frac{d}{dr}\ B(r),\ (2,\ 2,\ 3)=0,\ (2,\ 2,\ 4)=0,\ (2,\ 3,\ 1)=0,\ (2,\ 3,\ 2)=0,\ (2,\ 3,\ 3)=-\frac{r}{e^{2\ B(r)}},\ (2,\ 3,\ 4)=0,\ (2,\ 4,\ 1)=0,\ (2,\ 4,\ 2)=0,\ (2,\ 4,\ 3)=0,\ (2,\ 4,\ 4)=-\frac{r\ \sin(\theta)^2}{e^{2\ B(r)}},\ (3,\ 1,\ 1)=0,\ (3,\ 1,\ 2)=0,\ (3,\ 1,\ 3)=0,\ (3,\ 1,\ 4)=0,\ (3,\ 2,\ 1)=0,\ (3,\ 2,\ 2)=0,\ (3,\ 2,\ 3)=\frac{1}{r},\ (3,\ 2,\ 4)=0,\ (3,\ 3,\ 1)=0,\ (3,\ 3,\ 2)=\frac{1}{r},\ (3,\ 3,\ 3)=0,\ (3,\ 3,\ 4)=0,\ (3,\ 4,\ 1)=0,\ (3,\ 4,\ 2)=0,\ (3,\ 4,\ 3)=0,\ (3,\ 4,\ 4)=-\sin(\theta)\ \cos(\theta),\ (4,\ 1,\ 1)=0,\ (4,\ 1,\ 2)=0,\ (4,\ 1,\ 3)=0,\ (4,\ 1,\ 4)=0,\ (4,\ 2,\ 1)=0,\ (4,\ 2,\ 2)=0,\ (4,\ 2,\ 3)=0,\ (4,\ 2,\ 4)=\frac{1}{r},\ (4,\ 3,\ 1)=0,\ (4,\ 3,\ 2)=0,\ (4,\ 3,\ 3)=0,\ (4,\ 3,\ 4)=\frac{\cos(\theta)}{\sin(\theta)},\ (4,\ 4,\ 1)=0,\ (4,\ 4,\ 2)=\frac{1}{r},\ (4,\ 4,\ 3)=\frac{\cos(\theta)}{\sin(\theta)},\ (4,\ 4,\ 4)=0\right]\right)\right]\right)$$

Robertson-Walker-Space

```
restart
with(tensor) :
coord := [r, θ, ϕ]
```

$$[r, \theta, \phi]$$

```
g_compts := array(symmetric, sparse, 1..3, 1..3)
```

$$array(symmetric, sparse, 1..3, 1..3, [\,])$$

$g\_compts_{1,\,1} := -\exp(2\cdot B(t, r))$: $g\_compts_{2,\,2} := -r^2$:

$g\_compts_{3,\,3} := -r^2\sin(\theta)^2$: $g := create([-1, -1], eval(g\_compts))$

$$table\left(\left[index\_char = [-1, -1],\ compts = \begin{bmatrix} -e^{2\,B(t,\,r)} & 0 & 0 \\ 0 & -r^2 & 0 \\ 0 & 0 & -r^2\sin(\theta)^2 \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg')
```

$$table\left(\left[index\_char = [1, 1],\ compts = \begin{bmatrix} -\frac{1}{e^{2\,B(t,\,r)}} & 0 & 0 \\ 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & -\frac{1}{r^2\sin(\theta)^2} \end{bmatrix}\right]\right)$$

```
D1g := d1metric(g, coord) : D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1, 3, 1, 3] = -\left(\frac{\partial}{\partial r} B(t, r)\right) r\sin(\theta)^2,\ [1, 2, 1, 2] = -\left(\frac{\partial}{\partial r} B(t, r)\right) r,\ [2, 3, 2, 3] = \frac{r^2\left(\cos(\theta)^2 e^{2\,B(t,\,r)} + 1 - \cos(\theta)^2 - e^{2\,B(t,\,r)}\right)}{e^{2\,B(t,\,r)}}\right]$$

```
RICCI := Ricci(ginv, RMN)
```

$$table\left(\left[index\_char = [-1, -1],\ compts = \left[\left[-\frac{2\left(\frac{\partial}{\partial r} B(t, r)\right)}{r}, 0, 0\right], \left[0, -\frac{\left(\frac{\partial}{\partial r} B(t, r)\right) r + e^{2\,B(t,\,r)} - 1}{e^{2\,B(t,\,r)}}, 0\right], \left[0, 0, \frac{1}{e^{2\,B(t,\,r)}}\left(\cos(\theta)^2\left(\frac{\partial}{\partial r} B(t, r)\right) r + \cos(\theta)^2 e^{2\,B(t,\,r)} - \cos(\theta)^2 - \left(\frac{\partial}{\partial r} B(t, r)\right) r - e^{2\,B(t,\,r)} + 1\right)\right]\right]\right]\right)$$

$RS := Ricciscalar(ginv, RICCI)$

$$table\left(\left[index\_char = [\,],\ compts = \frac{2\left(2\left(\frac{\partial}{\partial r} B(t, r)\right) r + e^{2 B(t, r)} - 1\right)}{e^{2 B(t, r)} r^2}\right]\right)$$

$Estn := Einstein(g, RICCI, RS)$

$$table\left(\left[index\_char = [-1, -1],\ compts = \begin{bmatrix} \frac{e^{2 B(t, r)} - 1}{r^2} & 0 & 0 \\ 0 & \frac{\left(\frac{\partial}{\partial r} B(t, r)\right) r}{e^{2 B(t, r)}} & 0 \\ 0 & 0 & -\frac{\left(\frac{\partial}{\partial r} B(t, r)\right) r \left(-1 + \cos(\theta)^2\right)}{e^{2 B(t, r)}} \end{bmatrix}\right]\right)$$

$Cf2 := Christoffel2(ginv, Cf1)$

$$table\left(\left[index\_char = [1, -1, -1],\ compts = ARRAY\left(cf2, [1..3, 1..3, 1..3], \left[(1, 1, 1) = \frac{\partial}{\partial r} B(t, r), (1, 1, 2) = 0, (1, 1, 3) = 0, (1, 2, 1) = 0, (1, 2, 2) = -\frac{r}{e^{2 B(t, r)}}, (1, 2, 3) = 0, (1, 3, 1) = 0, (1, 3, 2) = 0, (1, 3, 3) = -\frac{r \sin(\theta)^2}{e^{2 B(t, r)}}, (2, 1, 1) = 0, (2, 1, 2) = \frac{1}{r}, (2, 1, 3) = 0, (2, 2, 1) = \frac{1}{r}, (2, 2, 2) = 0, (2, 2, 3) = 0, (2, 3, 1) = 0, (2, 3, 2) = 0, (2, 3, 3) = -\sin(\theta) \cos(\theta), (3, 1, 1) = 0, (3, 1, 2) = 0, (3, 1, 3) = \frac{1}{r}, (3, 2, 1) = 0, (3, 2, 2) = 0, (3, 2, 3) = \frac{\cos(\theta)}{\sin(\theta)}, (3, 3, 1) = \frac{1}{r}, (3, 3, 2) = \frac{\cos(\theta)}{\sin(\theta)}, (3, 3, 3) = 0\right]\right)\right]\right)$$

RW metric

$restart$

$with(tensor):$

$coord := [t, r, \theta, \phi]$

$$[t, r, \theta, \phi]$$

$g\_compts := array(symmetric, sparse, 1..4, 1..4)$

$$array(symmetric, sparse, 1..4, 1..4, [\,])$$

$g\_compts_{1,1} := 1 : \; g\_compts_{2,2} := \frac{-a(t)^2}{1 - k \cdot r^2} :$

$g\_compts_{3,3} := -a(t)^2 \cdot r^2 : \; g\_compts_{4,4} := -a(t)^2 \cdot r^2 \sin(\theta)^2 : \; g := create([-1, -1], eval(g\_compts))$

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a(t)^2}{-k\,r^2 + 1} & 0 & 0 \\ 0 & 0 & -a(t)^2\,r^2 & 0 \\ 0 & 0 & 0 & -a(t)^2\,r^2\,\sin(\theta)^2 \end{bmatrix}\right]\right)$$

$ginv := invert(g, \text{'}detg\text{'})$

$$table\left(\left[index\_char = [1, 1], compts = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{k\,r^2 - 1}{a(t)^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2\,a(t)^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sin(\theta)^2\,r^2\,a(t)^2} \end{bmatrix}\right]\right)$$

$D1g := d1metric(g, coord) : \; D2g := d2metric(D1g, coord) :$

$Cf1 := Christoffel1(D1g) :$

$RMN := Riemann(ginv, D2g, Cf1) :$

$RMNc := get\_compts(RMN) :$

```
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
 [ indices(RMNc)] );
```

$$\left[[1,4,1,4] = a(t)\, r^2 \sin(\theta)^2 \left(\frac{d^2}{dt^2} a(t)\right), [1,3,1,3] = a(t)\, r^2 \left(\frac{d^2}{dt^2} a(t)\right), [2,3,2,3] = \frac{r^2\, a(t)^2 \left(\left(\frac{d}{dt} a(t)\right)^2 + k\right)}{k\, r^2 - 1}, [1,2,1,2] = -\frac{a(t) \left(\frac{d^2}{dt^2} a(t)\right)}{k\, r^2 - 1}, [3,4,3,4] = a(t)^2\, r^4 \left(\cos(\theta)^2 \left(\frac{d}{dt} a(t)\right)^2 + \cos(\theta)^2\, k - \left(\frac{d}{dt} a(t)\right)^2 - k\right), [2,4,2,4] = \frac{\sin(\theta)^2\, r^2\, a(t)^2 \left(\left(\frac{d}{dt} a(t)\right)^2 + k\right)}{k\, r^2 - 1}\right]$$

$RICCI := Ricci(ginv, RMN)$

$$table\left(\left[index\_char = [-1,-1],\ compts = \left[\left[\frac{3 \left(\frac{d^2}{dt^2} a(t)\right)}{a(t)}, 0, 0, 0\right], \left[0, \frac{a(t) \left(\frac{d^2}{dt^2} a(t)\right) + 2 \left(\frac{d}{dt} a(t)\right)^2 + 2\, k}{k\, r^2 - 1}, 0, 0\right], \left[0, 0, -a(t)\, r^2 \left(\frac{d^2}{dt^2} a(t)\right) - 2 \left(\frac{d}{dt} a(t)\right)^2 r^2 - 2\, k\, r^2, 0\right], \left[0, 0, 0, r^2 \left(\left(\frac{d^2}{dt^2} a(t)\right) \cos(\theta)^2\, a(t) + 2 \cos(\theta)^2 \left(\frac{d}{dt} a(t)\right)^2 + 2 \cos(\theta)^2\, k - a(t) \left(\frac{d^2}{dt^2} a(t)\right) - 2 \left(\frac{d}{dt} a(t)\right)^2 - 2\, k\right)\right]\right]\right]\right)$$

$RS := Ricciscalar(ginv, RICCI)$

$$table\left(\left[index\_char = [\,],\ compts = \frac{6 \left(a(t) \left(\frac{d^2}{dt^2} a(t)\right) + \left(\frac{d}{dt} a(t)\right)^2 + k\right)}{a(t)^2}\right]\right)$$

$Estn := Einstein(g, RICCI, RS)$

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\left[\left[-\frac{3\left(\left(\frac{d}{dt}\ a(t)\right)^2+k\right)}{a(t)^2},\ 0,\ 0,\ 0\right],\right.\right.\right.$$

$$\left[0,\ -\frac{2\ a(t)\ \left(\frac{d^2}{dt^2}\ a(t)\right)+\left(\frac{d}{dt}\ a(t)\right)^2+k}{k\ r^2-1},\ 0,\ 0\right],$$

$$\left[0,\ 0,\ 2\ a(t)\ r^2\left(\frac{d^2}{dt^2}\ a(t)\right)+\left(\frac{d}{dt}\ a(t)\right)^2\ r^2+k\ r^2,\ 0\right],$$

$$\left[0,\ 0,\ 0,\ -r^2\left(2\left(\frac{d^2}{dt^2}\ a(t)\right)\cos(\theta)^2\ a(t)+\cos(\theta)^2\left(\frac{d}{dt}\ a(t)\right)^2+\cos(\theta)^2\ k\right.\right.$$

$$\left.\left.\left.\left.\left.-2\ a(t)\left(\frac{d^2}{dt^2}\ a(t)\right)-\left(\frac{d}{dt}\ a(t)\right)^2-k\right)\right]\right]\right]\right)$$

$Cf2 := Christoffel2(ginv,\ Cf1)$

$$table\left(\left[index\_char=[1, -1, -1], compts=ARRAY\left(cf2, [1..4, 1..4, 1..4], \left[(1, 1, 1)=0, (1, 1, 2)=0, (1, 1, 3)=0, (1, 1, 4)=0, (1, 2, 1)=0, (1, 2, 2)= -\frac{a(t)\left(\frac{d}{dt} a(t)\right)}{k r^2-1}, (1, 2, 3)=0, (1, 2, 4)=0, (1, 3, 1)=0, (1, 3, 2)=0, (1, 3, 3)=a(t) r^2 \left(\frac{d}{dt} a(t)\right), (1, 3, 4)=0, (1, 4, 1)=0, (1, 4, 2)=0, (1, 4, 3)=0, (1, 4, 4)=a(t) r^2 \sin(\theta)^2 \left(\frac{d}{dt} a(t)\right), (2, 1, 1)=0, (2, 1, 2)=\frac{\frac{d}{dt} a(t)}{a(t)}, (2, 1, 3)=0, (2, 1, 4)=0, (2, 2, 1)=\frac{\frac{d}{dt} a(t)}{a(t)}, (2, 2, 2)=-\frac{k r}{k r^2-1}, (2, 2, 3)=0, (2, 2, 4)=0, (2, 3, 1)=0, (2, 3, 2)=0, (2, 3, 3)=\left(k r^2-1\right) r, (2, 3, 4)=0, (2, 4, 1)=0, (2, 4, 2)=0, (2, 4, 3)=0, (2, 4, 4)=\left(k r^2-1\right) r \sin(\theta)^2, (3, 1, 1)=0, (3, 1, 2)=0, (3, 1, 3)=\frac{\frac{d}{dt} a(t)}{a(t)}, (3, 1, 4)=0, (3, 2, 1)=0, (3, 2, 2)=0, (3, 2, 3)=\frac{1}{r}, (3, 2, 4)=0, (3, 3, 1)=\frac{\frac{d}{dt} a(t)}{a(t)}, (3, 3, 2)=\frac{1}{r}, (3, 3, 3)=0, (3, 3, 4)=0, (3, 4, 1)=0, (3, 4, 2)=0, (3, 4, 3)=0, (3, 4, 4)=-\sin(\theta) \cos(\theta), (4, 1, 1)=0, (4, 1, 2)=0, (4, 1, 3)=0, (4, 1, 4)=\frac{\frac{d}{dt} a(t)}{a(t)}, (4, 2, 1)=0, (4, 2, 2)=0, (4, 2, 3)=0, (4, 2, 4)=\frac{1}{r}, (4, 3, 1)=0, (4, 3, 2)=0, (4, 3, 3)=0, (4, 3, 4)=\frac{\cos(\theta)}{\sin(\theta)}, (4, 4, 1)=\frac{\frac{d}{dt} a(t)}{a(t)}, (4, 4, 2)=\frac{1}{r}, (4, 4, 3)=\frac{\cos(\theta)}{\sin(\theta)}, (4, 4, 4)=0\right]\right)\right]\right)$$

four-dim diagonal metric

```
> restart
with(tensor) :
coord := [t, r, θ, ϕ]
```

$$[t, r, \theta, \phi]$$

```
g_compts := array(symmetric, sparse, 1..4, 1..4) :
g_compts[1, 1] := f(t, r) :   g_compts[2, 2] := -G(t, r) :
g_compts[3, 3] := -r^2 :  g_compts[4, 4] := -r^2 · sin(θ)^2 :  g := create([-1, -1], eval(g_compts))
```

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} f(t, r) & 0 & 0 & 0 \\ 0 & -G(t, r) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg') :
D1g := d1metric(g, coord) :  D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
 [ indices(RMNc)] );
```

$$\left[[1,3,2,3] = -\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial t} G(t,r)\right) r}{G(t,r)},\ [3,4,3,4] = \frac{r^2 \left(\cos(\theta)^2 G(t,r) + 1 - \cos(\theta)^2 - G(t,r)\right)}{G(t,r)},\ [2,3,2,3] = -\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial r} G(t,r)\right) r}{G(t,r)},\ [1,4,2,4] = -\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial t} G(t,r)\right) r \sin(\theta)^2}{G(t,r)},\ [1,3,1,3] = -\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial r} f(t,r)\right) r}{G(t,r)},\ [1,4,1,4] = -\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial r} f(t,r)\right) r \sin(\theta)^2}{G(t,r)},\ [1,2,1,2] = -\frac{1}{4}\,\frac{1}{f(t,r)\, G(t,r)}\left(2\left(\frac{\partial^2}{\partial r^2} f(t,r)\right) f(t,r)\, G(t,r) - \left(\frac{\partial}{\partial r} f(t,r)\right)^2 G(t,r) - \left(\frac{\partial}{\partial r} f(t,r)\right)\left(\frac{\partial}{\partial r} G(t,r)\right) f(t,r) + \left(\frac{\partial}{\partial t} f(t,r)\right)\left(\frac{\partial}{\partial t} G(t,r)\right) G(t,r) - 2\left(\frac{\partial^2}{\partial t^2} G(t,r)\right) f(t,r)\, G(t,r) + \left(\frac{\partial}{\partial t} G(t,r)\right)^2 f(t,r)\right),\ [2,4,2,4] = -\frac{1}{2}\,\frac{\left(\frac{\partial}{\partial r} G(t,r)\right) r \sin(\theta)^2}{G(t,r)} \right]$$

$RICCI := Ricci(ginv, RMN)$

$$table\left(\left[index\_char=[-1,-1],\ compts=\left[\left[-\frac{1}{4}\,\frac{1}{G(t,r)^2\,f(t,r)\,r}\left(2\left(\frac{\partial^2}{\partial r^2}f(t,r)\right)G(t,r)\,f(t,r)\,r-\left(\frac{\partial}{\partial r}f(t,r)\right)^2G(t,r)\,r-\left(\frac{\partial}{\partial r}f(t,r)\right)\left(\frac{\partial}{\partial r}G(t,r)\right)f(t,r)\,r+\left(\frac{\partial}{\partial t}f(t,r)\right)\left(\frac{\partial}{\partial t}G(t,r)\right)G(t,r)\,r-2\left(\frac{\partial^2}{\partial t^2}G(t,r)\right)G(t,r)\,f(t,r)\,r+\left(\frac{\partial}{\partial t}G(t,r)\right)^2f(t,r)\,r+4\left(\frac{\partial}{\partial r}f(t,r)\right)G(t,r)\,f(t,r)\right),\ -\frac{\frac{\partial}{\partial t}G(t,r)}{r\,G(t,r)},\ 0,\ 0\right],\right.\right.\right.$$

$$\left[-\frac{\frac{\partial}{\partial t}G(t,r)}{r\,G(t,r)},\ \frac{1}{4}\,\frac{1}{f(t,r)^2\,G(t,r)\,r}\left(2\left(\frac{\partial^2}{\partial r^2}f(t,r)\right)G(t,r)\,f(t,r)\,r-\left(\frac{\partial}{\partial r}f(t,r)\right)^2G(t,r)\,r-\left(\frac{\partial}{\partial r}f(t,r)\right)\left(\frac{\partial}{\partial r}G(t,r)\right)f(t,r)\,r+\left(\frac{\partial}{\partial t}f(t,r)\right)\left(\frac{\partial}{\partial t}G(t,r)\right)G(t,r)\,r-2\left(\frac{\partial^2}{\partial t^2}G(t,r)\right)G(t,r)\,f(t,r)\,r+\left(\frac{\partial}{\partial t}G(t,r)\right)^2f(t,r)\,r-4\left(\frac{\partial}{\partial r}G(t,r)\right)f(t,r)^2\right),\ 0,\ 0\right],$$

$$\left[0,\ 0,\ \frac{1}{2}\,\frac{1}{G(t,r)^2\,f(t,r)}\left(\left(\frac{\partial}{\partial r}f(t,r)\right)r\,G(t,r)-\left(\frac{\partial}{\partial r}G(t,r)\right)r\,f(t,r)-2\,G(t,r)^2\,f(t,r)+2\,f(t,r)\,G(t,r)\right),\ 0\right],$$

$$\left.\left.\left.\left[0,\ 0,\ 0,\ -\frac{1}{2}\,\frac{1}{G(t,r)^2\,f(t,r)}\left(\left(\frac{\partial}{\partial r}f(t,r)\right)\cos(\theta)^2\,G(t,r)\,r-\cos(\theta)^2\left(\frac{\partial}{\partial r}G(t,r)\right)f(t,r)\,r-2\cos(\theta)^2\,G(t,r)^2\,f(t,r)+2\cos(\theta)^2\,G(t,r)\,f(t,r)-\left(\frac{\partial}{\partial r}f(t,r)\right)r\,G(t,r)+\left(\frac{\partial}{\partial r}G(t,r)\right)r\,f(t,r)+2\,G(t,r)^2\,f(t,r)-2\,f(t,r)\,G(t,r)\right)\right]\right]\right]\right)$$

$RS := Ricciscalar(ginv, RICCI)$

$$table\left(\left[index\_char=[\,],\ compts=-\frac{1}{2}\,\frac{1}{f(t,r)^2\,G(t,r)^2\,r^2}\left(2\left(\frac{\partial^2}{\partial r^2}f(t,r)\right)G(t,r)\,f(t,r)\,r^2-\left(\frac{\partial}{\partial r}f(t,r)\right)^2G(t,r)\,r^2-\left(\frac{\partial}{\partial r}f(t,r)\right)\left(\frac{\partial}{\partial r}G(t,r)\right)f(t,r)\,r^2+\left(\frac{\partial}{\partial t}f(t,r)\right)\left(\frac{\partial}{\partial t}G(t,r)\right)G(t,r)\,r^2-2\left(\frac{\partial^2}{\partial t^2}G(t,r)\right)G(t,r)\,f(t,r)\,r^2+\left(\frac{\partial}{\partial t}G(t,r)\right)^2f(t,r)\,r^2+4\left(\frac{\partial}{\partial r}f(t,r)\right)G(t,r)\,f(t,r)\,r-4\left(\frac{\partial}{\partial r}G(t,r)\right)f(t,r)^2\,r-4\,G(t,r)^2\,f(t,r)^2+4\,f(t,r)^2\,G(t,r)\right)\right]\right)$$

$Estn := Einstein(g, RICCI, RS)$

$$table\left(\left[index\_char = [-1, -1],\ compts = \left[\left[-\frac{f(t, r)\left(\left(\frac{\partial}{\partial r} G(t, r)\right) r + G(t, r)^2 - G(t, r)\right)}{r^2\, G(t, r)^2},\ -\frac{\frac{\partial}{\partial t} G(t, r)}{r\, G(t, r)},\ 0,\ 0\right],\right.\right.\right.$$

$$\left[-\frac{\frac{\partial}{\partial t} G(t, r)}{r\, G(t, r)},\ -\frac{\left(\frac{\partial}{\partial r} f(t, r)\right) r - f(t, r)\, G(t, r) + f(t, r)}{f(t, r)\, r^2},\ 0,\ 0\right],$$

$$\left[0,\ 0,\ -\frac{1}{4}\, \frac{1}{G(t, r)^2\, f(t, r)^2}\left(r\left(2\left(\frac{\partial^2}{\partial r^2} f(t, r)\right) G(t, r)\, f(t, r)\, r - \left(\frac{\partial}{\partial r} f(t, r)\right)^2 G(t, r)\, r - \left(\frac{\partial}{\partial r} f(t, r)\right)\left(\frac{\partial}{\partial r} G(t, r)\right) f(t, r)\, r + \left(\frac{\partial}{\partial t} f(t, r)\right)\left(\frac{\partial}{\partial t} G(t, r)\right) G(t, r)\, r - 2\left(\frac{\partial^2}{\partial t^2} G(t, r)\right) G(t, r)\, f(t, r)\, r + \left(\frac{\partial}{\partial t} G(t, r)\right)^2 f(t, r)\, r + 2\left(\frac{\partial}{\partial r} f(t, r)\right) G(t, r)\, f(t, r) - 2\left(\frac{\partial}{\partial r} G(t, r)\right) f(t, r)^2\right)\right),\ 0\right],$$

$$\left[0,\ 0,\ 0,\ \frac{1}{4}\, \frac{1}{G(t, r)^2\, f(t, r)^2}\left(r\left(2\left(\frac{\partial^2}{\partial r^2} f(t, r)\right) \cos(\theta)^2\, G(t, r)\, f(t, r)\, r - \left(\frac{\partial}{\partial r} f(t, r)\right)^2 \cos(\theta)^2\, G(t, r)\, r - \left(\frac{\partial}{\partial r} f(t, r)\right) \cos(\theta)^2 \left(\frac{\partial}{\partial r} G(t, r)\right) f(t, r)\, r + \left(\frac{\partial}{\partial t} f(t, r)\right) \cos(\theta)^2 \left(\frac{\partial}{\partial t} G(t, r)\right) G(t, r)\, r - 2\left(\frac{\partial^2}{\partial t^2} G(t, r)\right) \cos(\theta)^2\, G(t, r)\, f(t, r)\, r + \cos(\theta)^2 \left(\frac{\partial}{\partial t} G(t, r)\right)^2 f(t, r)\, r + 2\left(\frac{\partial}{\partial r} f(t, r)\right) \cos(\theta)^2\, G(t, r)\, f(t, r) - 2\cos(\theta)^2 \left(\frac{\partial}{\partial r} G(t, r)\right) f(t, r)^2 - 2\left(\frac{\partial^2}{\partial r^2} f(t, r)\right) G(t, r)\, f(t, r)\, r + \left(\frac{\partial}{\partial r} f(t, r)\right)^2 G(t, r)\, r + \left(\frac{\partial}{\partial r} f(t, r)\right)\left(\frac{\partial}{\partial r} G(t, r)\right) f(t, r)\, r - \left(\frac{\partial}{\partial t} f(t, r)\right)\left(\frac{\partial}{\partial t} G(t, r)\right) G(t, r)\, r + 2\left(\frac{\partial^2}{\partial t^2} G(t, r)\right) G(t, r)\, f(t, r)\, r - \left(\frac{\partial}{\partial t} G(t, r)\right)^2 f(t, r)\, r - 2\left(\frac{\partial}{\partial r} f(t, r)\right) G(t, r)\, f(t, r) + 2\left(\frac{\partial}{\partial r} G(t, r)\right) f(t, r)^2\right)\right)\right]\left.\left.\left.\right]\right]\right)$$

$Cf2 := Christoffel2(ginv, Cf1)$

$$table\left(\left[index\_char=[1,-1,-1],\ compts=ARRAY\left(cf2,[1..4,1..4,1..4],\left[(1,1,1)=\frac{1}{2}\frac{\frac{\partial}{\partial t}f(t,r)}{f(t,r)},\ (1,1,2)=\frac{1}{2}\frac{\frac{\partial}{\partial r}f(t,r)}{f(t,r)},\ (1,1,3)=0,\ (1,1,4)=0,\ (1,2,1)=\frac{1}{2}\frac{\frac{\partial}{\partial r}f(t,r)}{f(t,r)},\ (1,2,2)=\frac{1}{2}\frac{\frac{\partial}{\partial t}G(t,r)}{f(t,r)},\ (1,2,3)=0,\ (1,2,4)=0,\ (1,3,1)=0,\ (1,3,2)=0,\ (1,3,3)=0,\ (1,3,4)=0,\ (1,4,1)=0,\ (1,4,2)=0,\ (1,4,3)=0,\ (1,4,4)=0,\ (2,1,1)=\frac{1}{2}\frac{\frac{\partial}{\partial r}f(t,r)}{G(t,r)},\ (2,1,2)=\frac{1}{2}\frac{\frac{\partial}{\partial t}G(t,r)}{G(t,r)},\ (2,1,3)=0,\ (2,1,4)=0,\ (2,2,1)=\frac{1}{2}\frac{\frac{\partial}{\partial t}G(t,r)}{G(t,r)},\ (2,2,2)=\frac{1}{2}\frac{\frac{\partial}{\partial r}G(t,r)}{G(t,r)},\ (2,2,3)=0,\ (2,2,4)=0,\ (2,3,1)=0,\ (2,3,2)=0,\ (2,3,3)=-\frac{r}{G(t,r)},\ (2,3,4)=0,\ (2,4,1)=0,\ (2,4,2)=0,\ (2,4,3)=0,\ (2,4,4)=-\frac{r\sin(\theta)^2}{G(t,r)},\ (3,1,1)=0,\ (3,1,2)=0,\ (3,1,3)=0,\ (3,1,4)=0,\ (3,2,1)=0,\ (3,2,2)=0,\ (3,2,3)=\frac{1}{r},\ (3,2,4)=0,\ (3,3,1)=0,\ (3,3,2)=\frac{1}{r},\ (3,3,3)=0,\ (3,3,4)=0,\ (3,4,1)=0,\ (3,4,2)=0,\ (3,4,3)=0,\ (3,4,4)=-\sin(\theta)\cos(\theta),\ (4,1,1)=0,\ (4,1,2)=0,\ (4,1,3)=0,\ (4,1,4)=0,\ (4,2,1)=0,\ (4,2,2)=0,\ (4,2,3)=0,\ (4,2,4)=\frac{1}{r},\ (4,3,1)=0,\ (4,3,2)=0,\ (4,3,3)=0,\ (4,3,4)=\frac{\cos(\theta)}{\sin(\theta)},\ (4,4,1)=0,\ (4,4,2)=\frac{1}{r},\ (4,4,3)=\frac{\cos(\theta)}{\sin(\theta)},\ (4,4,4)=0\right]\right)\right]\right)$$

Ads

```
> ginv := invert(g, 'detg')
```

$$table\left(\left[index\_char = [1, 1], compts = \begin{bmatrix} -\frac{r^2}{-r^4 - r^2 + \mu} & 0 & 0 & 0 & 0 \\ 0 & \frac{-r^4 - r^2 + \mu}{r^2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2 \sin(\psi)^2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{r^2 \sin(\psi)^2 \sin(\theta)^2} \end{bmatrix}\right]\right)$$

```
D1g := d1metric(g, coord) : D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1,2,1,2]=\frac{-r^4+3\mu}{r^4},\ [2,3,2,3]=-\frac{r^4+\mu}{-r^4-r^2+\mu},\ [1,5,1,5]=\frac{(-r^4-r^2+\mu)\,(r^4+\mu)\,\sin(\psi)^2\,\sin(\theta)^2}{r^4},\ [1,4,1,4]=\frac{(-r^4-r^2+\mu)\,(r^4+\mu)\,\sin(\psi)^2}{r^4},\ [3,4,3,4]=-\sin(\psi)^2\,(-r^4+\mu),\ [4,5,4,5]=-\sin(\psi)^2\,\big(-\cos(\theta)^2\cos(\psi)^2 r^4+\cos(\theta)^2 r^4+\cos(\psi)^2 r^4+\cos(\theta)^2\cos(\psi)^2\mu-r^4-\cos(\theta)^2\mu-\cos(\psi)^2\mu+\mu\big),\ [2,4,2,4]=-\frac{(r^4+\mu)\,\sin(\psi)^2}{-r^4-r^2+\mu},\ [2,5,2,5]=-\frac{(r^4+\mu)\,\sin(\psi)^2\,\sin(\theta)^2}{-r^4-r^2+\mu},\ [1,3,1,3]=\frac{(-r^4-r^2+\mu)\,(r^4+\mu)}{r^4},\ [3,5,3,5]=-\sin(\theta)^2\,\sin(\psi)^2\,(-r^4+\mu)\right]$$

RICCI := *Ricci*(*ginv*, *RMN*)

$$\mathit{table}\left(\left[\mathit{index\_char}=[-1,-1],\ \mathit{compts}=\left[\left[\frac{4\,(-r^4-r^2+\mu)}{r^2},0,0,0,0\right],\left[0,-\frac{4\,r^2}{-r^4-r^2+\mu},0,0,0\right],\left[0,0,4\,r^2,0,0\right],\left[0,0,0,\frac{4\,r^2\,(\cos(\theta)^2\cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1)}{\sin(\theta)^2},0\right],\left[0,0,0,0,4\,r^2\,(\cos(\theta)^2\cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1)\right]\right]\right]\right)$$

RS := *Ricciscalar*(*ginv*, *RICCI*)

$$\mathit{table}\left(\left[\mathit{index\_char}=[\ ],\ \mathit{compts}=-\frac{20\,(\cos(\theta)^2\cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1)}{\sin(\psi)^2\,\sin(\theta)^2}\right]\right)$$

Estn := *Einstein*(*g*, *RICCI*, *RS*)

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\left[\left[-\frac{1}{r^2\ \sin(\psi)^2\ \sin(\theta)^2}\left(6\ \left(-\cos(\theta)^2\ \cos(\psi)^2\ r^4-\cos(\theta)^2\ \cos(\psi)^2\ r^2+\cos(\theta)^2\ r^4+\cos(\psi)^2\ r^4+\cos(\theta)^2\ \cos(\psi)^2\ \mu+r^2\ \cos(\theta)^2+r^2\ \cos(\psi)^2-r^4-\cos(\theta)^2\ \mu-\cos(\psi)^2\ \mu-r^2+\mu\right)\right),\ 0,\ 0,\ 0,\ 0\right],\right.\right.\right.$$

$$\left[0,\ \frac{6\ r^2\ \left(\cos(\theta)^2\ \cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1\right)}{\left(-r^4-r^2+\mu\right)\ \sin(\psi)^2\ \sin(\theta)^2},\ 0,\ 0,\ 0\right],$$

$$\left[0,\ 0,\ -\frac{6\ r^2\ \left(\cos(\theta)^2\ \cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1\right)}{\sin(\psi)^2\ \sin(\theta)^2},\ 0,\ 0\right],$$

$$\left[0,\ 0,\ 0,\ -\frac{6\ r^2\ \left(\cos(\theta)^2\ \cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1\right)}{\sin(\theta)^2},\ 0\right],$$

$$\left.\left.\left.\left[0,\ 0,\ 0,\ 0,\ -6\ r^2\ \left(\cos(\theta)^2\ \cos(\psi)^2-\cos(\theta)^2-\cos(\psi)^2+1\right)\right]\right]\right]\right)$$

$Cf2 := Christoffel2(ginv,\ Cf1)$

$$table\left(\left[index\_char=[1,-1,-1],\ compts=ARRAY\left(cf2,[1..5,1..5,1..5],\left[(1,1,1)=0,\ (1,1,2)=-\frac{r^4+\mu}{r\left(-r^4-r^2+\mu\right)},\ (1,1,3)=0,\ (1,1,4)=0,\ (1,1,5)=0,\right.\right.\right.\right.$$

$$(1,2,1)=-\frac{r^4+\mu}{r\left(-r^4-r^2+\mu\right)},\ (1,2,2)=0,\ (1,2,3)=0,\ (1,2,4)=0,\ (1,2,5)=0,\ (1,3,1)=0,\ (1,3,2)=0,\ (1,3,3)=0,\ (1,3,4)=0,\ (1,3,5)=0,\ (1,4,1)=0,$$

$$(1,4,2)=0,\ (1,4,3)=0,\ (1,4,4)=0,\ (1,4,5)=0,\ (1,5,1)=0,\ (1,5,2)=0,\ (1,5,3)=0,\ (1,5,4)=0,\ (1,5,5)=0,\ (2,1,1)=-\frac{\left(-r^4-r^2+\mu\right)\left(r^4+\mu\right)}{r^5},$$

$$(2,1,2)=0,\ (2,1,3)=0,\ (2,1,4)=0,\ (2,1,5)=0,\ (2,2,1)=0,\ (2,2,2)=\frac{r^4+\mu}{r\left(-r^4-r^2+\mu\right)},\ (2,2,3)=0,\ (2,2,4)=0,\ (2,2,5)=0,\ (2,3,1)=0,\ (2,3,2)=0,$$

$$(2,3,3)=\frac{-r^4-r^2+\mu}{r},\ (2,3,4)=0,\ (2,3,5)=0,\ (2,4,1)=0,\ (2,4,2)=0,\ (2,4,3)=0,\ (2,4,4)=\frac{\left(-r^4-r^2+\mu\right)\sin(\psi)^2}{r},\ (2,4,5)=0,\ (2,5,1)=0,\ (2,5,2)=0,$$

$$(2,5,3)=0,\ (2,5,4)=0,\ (2,5,5)=\frac{\left(-r^4-r^2+\mu\right)\sin(\psi)^2\sin(\theta)^2}{r},\ (3,1,1)=0,\ (3,1,2)=0,\ (3,1,3)=0,\ (3,1,4)=0,\ (3,1,5)=0,\ (3,2,1)=0,\ (3,2,2)=0,$$

$$(3,2,3)=\frac{1}{r},\ (3,2,4)=0,\ (3,2,5)=0,\ (3,3,1)=0,\ (3,3,2)=\frac{1}{r},\ (3,3,3)=0,\ (3,3,4)=0,\ (3,3,5)=0,\ (3,4,1)=0,\ (3,4,2)=0,\ (3,4,3)=0,\ (3,4,4)=-\sin(\psi)\cos(\psi),$$

$$(3,4,5)=0,\ (3,5,1)=0,\ (3,5,2)=0,\ (3,5,3)=0,\ (3,5,4)=0,\ (3,5,5)=-\sin(\psi)\sin(\theta)^2\cos(\psi),\ (4,1,1)=0,\ (4,1,2)=0,\ (4,1,3)=0,\ (4,1,4)=0,\ (4,1,5)=0,$$

$$(4,2,1)=0,\ (4,2,2)=0,\ (4,2,3)=0,\ (4,2,4)=\frac{1}{r},\ (4,2,5)=0,\ (4,3,1)=0,\ (4,3,2)=0,\ (4,3,3)=0,\ (4,3,4)=\frac{\cos(\psi)}{\sin(\psi)},\ (4,3,5)=0,\ (4,4,1)=0,\ (4,4,2)=\frac{1}{r},$$

$$(4,4,3)=\frac{\cos(\psi)}{\sin(\psi)},\ (4,4,4)=0,\ (4,4,5)=0,\ (4,5,1)=0,\ (4,5,2)=0,\ (4,5,3)=0,\ (4,5,4)=0,\ (4,5,5)=-\sin(\theta)\cos(\theta),\ (5,1,1)=0,\ (5,1,2)=0,\ (5,1,3)=0,\ (5,1,4)=0,\ (5,1,5)$$

2-d

```
restart
with(tensor) :
coord := [x, y]
```

$$[x, y]$$

```
g_compts := array(symmetric, sparse, 1..2, 1..2) :
g_compts[1, 1] := F(x, y) :   g_compts[2, 2] := -G(t, r) :
  g := create([-1, -1], eval(g_compts))
```

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} F(x, y) & 0 \\ 0 & -G(t, r) \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg') :
D1g := d1metric(g, coord) :  D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1, 2, 1, 2] = -\frac{1}{4} \frac{2 \left(\frac{\partial^2}{\partial y^2} F(x, y)\right) F(x, y) - \left(\frac{\partial}{\partial y} F(x, y)\right)^2}{F(x, y)}\right]$$

```
RICCI := Ricci(ginv, RMN)
```

$$table\left(\left[index\_char = [-1, -1], compts = \left[\left[-\frac{1}{4} \frac{2 \left(\frac{\partial^2}{\partial y^2} F(x, y)\right) F(x, y) - \left(\frac{\partial}{\partial y} F(x, y)\right)^2}{G(t, r) F(x, y)}, 0\right], \left[0, \frac{1}{4} \frac{2 \left(\frac{\partial^2}{\partial y^2} F(x, y)\right) F(x, y) - \left(\frac{\partial}{\partial y} F(x, y)\right)^2}{F(x, y)^2}\right]\right]\right]\right)$$

```
RS := Ricciscalar(ginv, RICCI)
```

$$table\left(\left[index\_char = [\,], compts = -\frac{1}{2} \frac{2 \left(\frac{\partial^2}{\partial y^2} F(x, y)\right) F(x, y) - \left(\frac{\partial}{\partial y} F(x, y)\right)^2}{F(x, y)^2 G(t, r)}\right]\right)$$

```
Estn := Einstein(g, RICCI, RS)
```

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right]\right)$$

```
Cf2 := Christoffel2(ginv, Cf1)
```

$$table\left(\left[index\_char = [1, -1, -1], compts = ARRAY\left(cf2, [1..2, 1..2, 1..2], \left[(1, 1, 1) = \frac{1}{2} \frac{\frac{\partial}{\partial x} F(x, y)}{F(x, y)}, (1, 1, 2) = \frac{1}{2} \frac{\frac{\partial}{\partial y} F(x, y)}{F(x, y)}, (1, 2, 1) = \frac{1}{2} \frac{\frac{\partial}{\partial y} F(x, y)}{F(x, y)}, (1, 2, 2) = 0, (2, 1, 1) = \frac{1}{2} \frac{\frac{\partial}{\partial y} F(x, y)}{G(t, r)}, (2, 1, 2) = 0, (2, 2, 1) = 0, (2, 2, 2) = 0\right]\right)\right]\right)$$

2-d general

```
restart
with(tensor) :
coord := [x, y]
```

$$[x, y]$$

```
g_compts := array(symmetric, sparse, 1..2, 1..2) :
g_compts[1, 1] := F(x, y) :   g_compts[2, 2] := -G(t, r) : g_compts[1, 2] := -K(t, r) :
g := create([-1, -1], eval(g_compts))
```

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} F(x, y) & -K(t, r) \\ -K(t, r) & -G(t, r) \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg') :
D1g := d1metric(g, coord) :  D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1, 2, 1, 2] = -\frac{1}{4} \frac{1}{F(x, y)\ G(t, r) + K(t, r)^2} \left(2 \left(\frac{\partial^2}{\partial y^2} F(x, y)\right) G(t, r)\ F(x, y) + 2 \left(\frac{\partial^2}{\partial y^2} F(x, y)\right) K(t, r)^2 - G(t, r) \left(\frac{\partial}{\partial y} F(x, y)\right)^2\right)\right]$$

```
RICCI := Ricci(ginv, RMN)
```

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\left[\left[-\frac{1}{4}\ \frac{1}{\left(F(x,\ y)\ G(t,\ r)+K(t,\ r)^2\right)^2}\left(F(x,\ y)\ \left(2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ G(t,\ r)\ F(x,\ y)+2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ K(t,\ r)^2-G(t,\ r)\ \left(\frac{\partial}{\partial y}\ F(x,\ y)\right)^2\right)\right),\right.\right.\right.$$

$$\frac{1}{4}\ \frac{1}{\left(F(x,\ y)\ G(t,\ r)+K(t,\ r)^2\right)^2}\left(K(t,\ r)\ \left(2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ G(t,\ r)\ F(x,\ y)+2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ K(t,\ r)^2-G(t,\ r)\ \left(\frac{\partial}{\partial y}\ F(x,\ y)\right)^2\right)\right)\Bigg],$$

$$\left[\frac{1}{4}\ \frac{1}{\left(F(x,\ y)\ G(t,\ r)+K(t,\ r)^2\right)^2}\left(K(t,\ r)\ \left(2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ G(t,\ r)\ F(x,\ y)+2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ K(t,\ r)^2-G(t,\ r)\ \left(\frac{\partial}{\partial y}\ F(x,\ y)\right)^2\right)\right),\right.$$

$$\left.\left.\left.\left.\frac{1}{4}\ \frac{1}{\left(F(x,\ y)\ G(t,\ r)+K(t,\ r)^2\right)^2}\left(G(t,\ r)\ \left(2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ G(t,\ r)\ F(x,\ y)+2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ K(t,\ r)^2-G(t,\ r)\ \left(\frac{\partial}{\partial y}\ F(x,\ y)\right)^2\right)\right)\right]\right]\right]\right)$$

$RS := Ricciscalar(ginv,\ RICCI)$

$$table\left(\left[index\_char=[\ ],\ compts=-\frac{1}{2}\ \frac{1}{\left(F(x,\ y)\ G(t,\ r)+K(t,\ r)^2\right)^2}\left(2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ G(t,\ r)\ F(x,\ y)+2\ \left(\frac{\partial^2}{\partial y^2}\ F(x,\ y)\right)\ K(t,\ r)^2-G(t,\ r)\ \left(\frac{\partial}{\partial y}\ F(x,\ y)\right)^2\right)\right]\right)$$

$Estn := Einstein(g,\ RICCI,\ RS)$

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\right]\right)$$

$Cf2 := Christoffel2(ginv,\ Cf1)$

$$table\left(\left[index\_char = [1, -1, -1], compts = ARRAY\left(cf2, [1..2, 1..2, 1..2], \left[(1, 1, 1) = \frac{1}{2}\,\frac{K(t, r)\left(\frac{\partial}{\partial y} F(x, y)\right) + G(t, r)\left(\frac{\partial}{\partial x} F(x, y)\right)}{F(x, y)\ G(t, r) + K(t, r)^2}, (1, 1, 2) = \frac{1}{2}\,\frac{G(t, r)\left(\frac{\partial}{\partial y} F(x, y)\right)}{F(x, y)\ G(t, r) + K(t, r)^2}, (1, 2, 1) = \frac{1}{2}\,\frac{G(t, r)\left(\frac{\partial}{\partial y} F(x, y)\right)}{F(x, y)\ G(t, r) + K(t, r)^2}, (1, 2, 2) = 0, (2, 1, 1) = \frac{1}{2}\,\frac{F(x, y)\left(\frac{\partial}{\partial y} F(x, y)\right) - K(t, r)\left(\frac{\partial}{\partial x} F(x, y)\right)}{F(x, y)\ G(t, r) + K(t, r)^2}, (2, 1, 2) = -\frac{1}{2}\,\frac{K(t, r)\left(\frac{\partial}{\partial y} F(x, y)\right)}{F(x, y)\ G(t, r) + K(t, r)^2}, (2, 2, 1) = -\frac{1}{2}\,\frac{K(t, r)\left(\frac{\partial}{\partial y} F(x, y)\right)}{F(x, y)\ G(t, r) + K(t, r)^2}, (2, 2, 2) = 0\right]\right)\right]\right)$$

Homework1

> $restart$

$with(tensor):$

$coord := [x, y]$

$$[x, y]$$

$g\_compts := array(symmetric, sparse, 1..2, 1..2):$

$g\_compts_{1,1} := -\frac{\left(1 - \frac{y^2}{R^2}\right)}{1 - \frac{(x^2 + y^2)}{R^2}}:\quad g\_compts_{2,2} := -\frac{\left(1 - \frac{y^2}{R^2}\right)}{1 - \frac{(x^2 + y^2)}{R^2}}: g\_compts_{1,2} := -\frac{\frac{x \cdot y}{R^2}}{1 - \frac{(x^2 + y^2)}{R^2}}:$

$g := create([-1, -1], eval(g\_compts))$

$$table\left(\left[index\_char = [-1, -1], compts = \begin{bmatrix} -\dfrac{1-\dfrac{y^2}{R^2}}{1-\dfrac{x^2+y^2}{R^2}} & -\dfrac{x\ y}{R^2\left(1-\dfrac{x^2+y^2}{R^2}\right)} \\ -\dfrac{x\ y}{R^2\left(1-\dfrac{x^2+y^2}{R^2}\right)} & -\dfrac{1-\dfrac{y^2}{R^2}}{1-\dfrac{x^2+y^2}{R^2}} \end{bmatrix}\right]\right)$$

```
ginv := invert(g, 'detg') :
D1g := d1metric(g, coord) : D2g := d2metric(D1g, coord) :
Cf1 := Christoffel1(D1g) :
RMN := Riemann(ginv, D2g, Cf1) :
RMNc := get_compts(RMN) :
map(proc(x) if RMNc[op(x)] ≠ 0 then x = RMNc[op(x)] else NULL end if end
    proc,
  [ indices(RMNc)] );
```

$$\left[[1, 2, 1, 2] = \left(3\ R^6\ x^2 - 2\ R^6\ y^2 - R^4\ x^4 - 8\ R^4\ x^2\ y^2 + 6\ R^4\ y^4 - R^2\ x^4\ y^2 + 7\ R^2\ x^2\ y^4 - 6\ R^2\ y^6 + 4\ x^4\ y^4 - 2\ x^2\ y^6 + 2\ y^8\right) \Big/ \left(\left(R^2 - x^2 - y^2\right)^3\ \left(R^4 - 2\ R^2\ y^2 - x^2\ y^2 + y^4\right)\right)\right]$$

```
RICCI := Ricci(ginv, RMN)
```

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\left[\left[\left((R^2-y^2)\ (3\ R^6\ x^2-2\ R^6\ y^2-R^4\ x^4 -8\ R^4\ x^2\ y^2+6\ R^4\ y^4-R^2\ x^4\ y^2+7\ R^2\ x^2\ y^4-6\ R^2\ y^6+4\ x^4\ y^4-2\ x^2\ y^6 +2\ y^8)\right)\Big/\left((R^2-x^2-y^2)^2\ (R^4-2\ R^2\ y^2-x^2\ y^2+y^4)^2\right),\ \left(x\ y\ (3\ R^6\ x^2 -2\ R^6\ y^2-R^4\ x^4-8\ R^4\ x^2\ y^2+6\ R^4\ y^4-R^2\ x^4\ y^2+7\ R^2\ x^2\ y^4-6\ R^2\ y^6 +4\ x^4\ y^4-2\ x^2\ y^6+2\ y^8)\right)\Big/\left((R^2-x^2-y^2)^2\ (R^4-2\ R^2\ y^2-x^2\ y^2+y^4)^2\right)\right], \left[\left(x\ y\ (3\ R^6\ x^2-2\ R^6\ y^2-R^4\ x^4-8\ R^4\ x^2\ y^2+6\ R^4\ y^4-R^2\ x^4\ y^2 +7\ R^2\ x^2\ y^4-6\ R^2\ y^6+4\ x^4\ y^4-2\ x^2\ y^6+2\ y^8)\right)\Big/\left((R^2-x^2-y^2)^2\ (R^4 -2\ R^2\ y^2-x^2\ y^2+y^4)^2\right),\ \left((R^2-y^2)\ (3\ R^6\ x^2-2\ R^6\ y^2-R^4\ x^4 -8\ R^4\ x^2\ y^2+6\ R^4\ y^4-R^2\ x^4\ y^2+7\ R^2\ x^2\ y^4-6\ R^2\ y^6+4\ x^4\ y^4-2\ x^2\ y^6 +2\ y^8)\right)\Big/\left((R^2-x^2-y^2)^2\ (R^4-2\ R^2\ y^2-x^2\ y^2+y^4)^2\right)\right]\right]\right]\right)$$

$RS := Ricciscalar(ginv,\ RICCI)$

$$table\left(\left[index\_char=[\,],\ compts=-\left(2\ (3\ R^6\ x^2-2\ R^6\ y^2-R^4\ x^4-8\ R^4\ x^2\ y^2 +6\ R^4\ y^4-R^2\ x^4\ y^2+7\ R^2\ x^2\ y^4-6\ R^2\ y^6+4\ x^4\ y^4-2\ x^2\ y^6+2\ y^8)\right)\Big/ \left((R^4-2\ R^2\ y^2-x^2\ y^2+y^4)^2\ (R^2-x^2-y^2)\right)\right]\right)$$

$Estn := Einstein(g,\ RICCI,\ RS)$

$$table\left(\left[index\_char=[-1,\ -1],\ compts=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\right]\right)$$

$Cf2 := Christoffel2(ginv,\ Cf1)$

$$table\left(\left[index\_char=[1,\ -1,\ -1],\ compts=ARRAY\left(cf2,\ [1..2,\ 1..2,\ 1..2],\ \left[(1,\ 1,\ 1)=\frac{(R^2-y^2)\ x\ (R^2-2\ y^2)}{(R^2-x^2-y^2)\ (R^4-2\ R^2\ y^2-x^2\ y^2+y^4)},\ (1,\ 1,\ 2)=0,\ (1,\ 2,\ 1)=0,\ (1,\ 2,\ 2)=-\frac{x\ (R^2\ x^2-2\ R^2\ y^2+2\ y^4)}{(R^2-x^2-y^2)\ (R^4-2\ R^2\ y^2-x^2\ y^2+y^4)},\ (2,\ 1,\ 1) =\frac{y\ (R^2-y^2)}{R^4-2\ R^2\ y^2-x^2\ y^2+y^4},\ (2,\ 1,\ 2)=\frac{x}{R^2-x^2-y^2},\ (2,\ 2,\ 1)=\frac{x}{R^2-x^2-y^2}, (2,\ 2,\ 2)=\frac{x^2\ y\ (R^2+x^2-3\ y^2)}{(R^2-x^2-y^2)\ (R^4-2\ R^2\ y^2-x^2\ y^2+y^4)}\right]\right)\right]\right)$$