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On the Evolution of Compact Objects

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A Comparison between the Size of WD and Earth

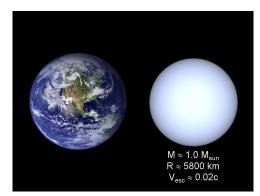


Figure: White Dwarf and Earth



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A Comparison between the Size of Neutron Stars and New York



Figure: Neutron Star and New York



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Thermodynamics of Compact Objects

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Landau Potential:
$$\Omega_{\rm p} = -kT \ln Z_G$$

Thermodynamical Potential: $\Omega = \int \Omega_{\rm p} \cdot \frac{g}{h^3} d^3 r d^3 p$
Particle Number Density: $n = \frac{1}{V} \int n_{\rm p} \cdot \frac{g}{h^3} d^3 r d^3 p$
Energy Density: $\rho_E = \frac{1}{V} \int \sqrt{({\rm p}c)^2 + (mc^2)^2} \cdot n_{\rm p} \frac{g}{h^3} d^3 r d^3 p$

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Thermodynamics of Compact Objects

Equation of States: White Dwarfs

$$\rho = \frac{m_B \nu_e}{3\pi^2 \lambda^3} x_F^3$$
$$P = \frac{mc^2}{3\pi^2 \lambda^3} \cdot \frac{3}{8} \left[x_F \sqrt{1 + x_F^2} \left(\frac{2}{3}x_F^2 - 1\right) + \operatorname{arc\,sinh}\left(x_F\right) \right]$$

where x_F is for quantization of particles' relativistic behavior, λ is particle's Compton wavelength.

$$x_F = \frac{p_F}{mc}$$
$$\lambda = \frac{\hbar}{mc}$$



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Equation of States: Neutron Stars

The following equation of states was formulated by Chandrasekhar and cited by Oppenheimer in his paper on neutron star theory in 1939.

$$\rho_E = \kappa(\sinh t - t)$$
$$P = \frac{\kappa}{3}(\sinh t + 3t - 8\sinh\frac{t}{2})$$

Remark

The above equation of state is merely a naive model. Modern theory uses far more complicated equation of state based on strong interactions between neutrons.

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Equilibrium of White Dwarfs

Equilibrium of Newtonian Theory

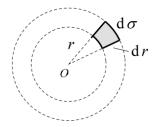


Figure: Sketch of White Dwarf

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP}{dr} = -\frac{G\rho(r)m(r)}{r^2}$$

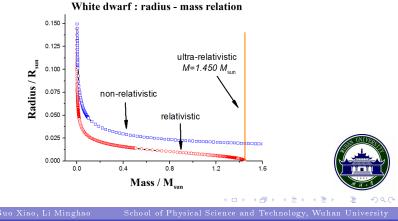
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Solution of White Dwarfs

Combine the equilibrium equation and equation of states, we are able to obtain the numerical solution of M-R relation.

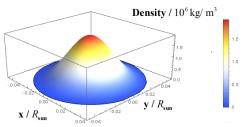




Equilibrium of White Dwarfs

Solution of White Dwarfs

The density distribution in the interior of white dwarfs is presented below



Density distribution in white dwarf

Figure: Density Distribution



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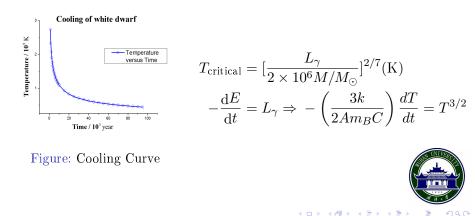
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Cooling Equations



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Equilibrium of Neutron Stars

Equation of Equilibrium

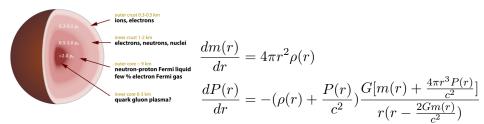


Figure: The possible interior structure of neutron star



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Equation of States: Neutron Stars

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Solution Corresponding to the Oppenheimer's EOS

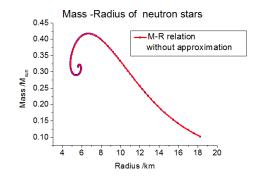


Figure: Numerical Solution of TOV Equation



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Solution Corresponding to the Approximation of Oppenheimer's EOS

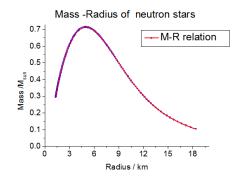


Figure: Numerical Solution of TOV Equation



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Equilibrium o			

A Qualitatively Illustration of the Increase of Maximum Mass

Equilibrium: Gravity against degeneracy pressure. Neutron's degeneracy pressure in the two cases are given by:

> $P = K \rho^{4/3}$ Relativistic Case $P = K \rho^{5/3}$ Non-Relativistic Case

Relativistic effects result in a weaker degeneracy pressure at a given density.



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Oppenheimer's Original Results

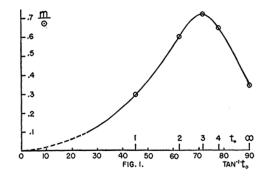


Figure: The solution of Oppenheimer and Volkoff.



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Cooling of Neutron Stars

In general, the cooling equation is:

$$\alpha T \frac{dT}{dt} = -bT^k - cT^2$$

The two terms in the equation correspond to neutrino emission and photon emission respectively.



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Figure: Hubble Telescope's Photo of Sirius A and B

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Figure: Hubble Telescope's Photo of NGC 2440: Pearl of a New White Dwarf

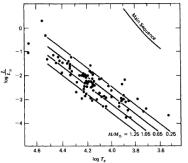


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White Dwarfs on H-R Diagram



According to *Stefan-Boltzmann Law*, we should have:

 $\log L \propto 4 \log T_e$

Therefore, white dwarfs are expected to occupy a narrow strap in H-R diagram.



Figure: White Dwarfs on H-R Diagram

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Measuring the Radii of White Dwarfs

According to energy conservation in radiation, we have:

$$F_{\nu}(\text{Measured}) = F_{\nu}(\text{Surface}) \frac{R^2}{D^2}$$

In this formula, all quantities except R can be directly or indirectly measured by observations.



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Astronomical Observation of White Dwarfs

Measuring the Masses of White Dwarfs Using Kepler's 3rd Law

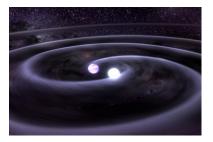


Figure: White Dwarfs Binary System

Invoking:

$$P = 2\pi \left(\frac{G(M_1 + M_2)}{a^3}\right)^{-1/2}$$
$$M_1 a_1 = M_2 a_2$$



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Measuring the Masses of White Dwarfs Using Gravitational Redshift

The formula is given below:

$$\frac{\Delta\lambda}{\lambda} = \frac{GM}{Rc^2}$$

Remark

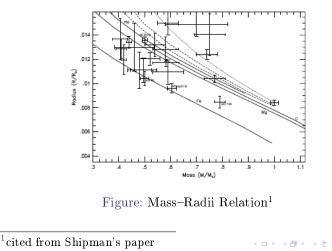
In order to distinguish redshift effects from Doppler effects, physicists usually use white dwarfs in wide binaries or common proper-motion pairs, for these white dwarfs' velocities can be measured accurately.

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Mass–Radius Relation as Experimental Result



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Figure: Crab Nebula: A Supernova Remnant



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Figure: Vela Pulsar: A Pulsar in a Supernova Remnant



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Measuring Neutron Stars' Masses Using Kepler's 3rd Law

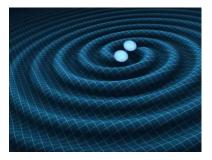


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Invoking:

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$$M_1 a_1 = M_2 a_2$$



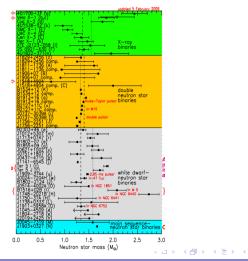
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Neutron Stars' Masses as Experimental Results





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