On the Evolution of Compact Objects

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2 Theory of White Dwarfs

3 Theory of Neutron Stars

4 Experimental Observation of Compact Objects

A Comparison between the Size of WD and Earth

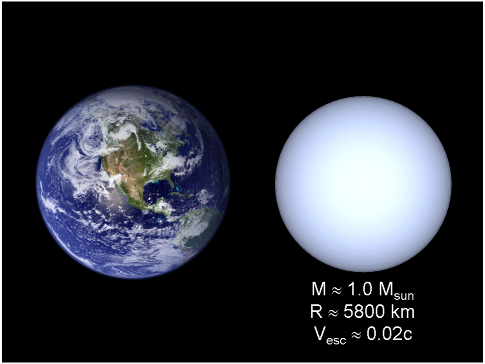

Figure: White Dwarf and Earth

A Comparison between the Size of Neutron Stars and New York

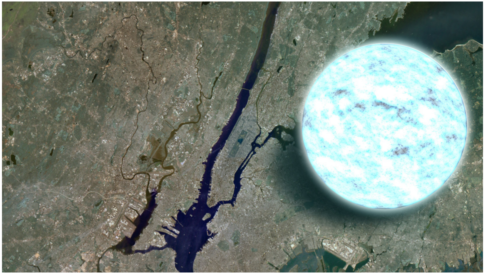

Figure: Neutron Star and New York

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- Equilibrium of White Dwarfs
- Cooling of White Dwarfs

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Landau Potential: $\Omega_{\mathrm{p}} = -kT\ln \mathrm{Z}_G$

Thermodynamical Potential: $\Omega = \int \Omega_{\mathrm{p}} \cdot \frac{g}{h^3} \mathrm{d}^3\mathrm{r}\mathrm{d}^3\mathrm{p}$

Particle Number Density: $n = \frac{1}{V} \int n_{\mathrm{p}} \cdot \frac{g}{h^3} \mathrm{d}^3\mathrm{r}\mathrm{d}^3\mathrm{p}$

Energy Density: $\rho_E = \frac{1}{V} \int \sqrt{(\mathrm{p}c)^2 + (mc^2)^2} \cdot n_{\mathrm{p}} \frac{g}{h^3} \mathrm{d}^3\mathrm{r}\mathrm{d}^3\mathrm{p}$

Equation of States: White Dwarfs

$$\rho = \frac{m_B \nu_e}{3\pi^2 \lambda^3} x_F^3$$

$$P = \frac{mc^2}{3\pi^2 \lambda^3} \cdot \frac{3}{8} \left[x_F \sqrt{1 + x_F^2} \left(\frac{2}{3} x_F^2 - 1 \right) + \operatorname{arc\,sinh}(x_F) \right]$$

where x_F is for quantization of particles' relativistic behavior, λ is particle's Compton wavelength.

$$x_F = \frac{p_F}{mc}$$

$$\lambda = \frac{\hbar}{mc}$$

Equation of States: Neutron Stars

The following equation of states was formulated by Chandrasekhar and cited by Oppenheimer in his paper on neutron star theory in 1939.

$$\rho_E = \kappa(\sinh t - t)$$

$$P = \frac{\kappa}{3}(\sinh t + 3t - 8\sinh\frac{t}{2})$$

Remark

The above equation of state is merely a naive model. Modern theory uses far more complicated equation of state based on strong interactions between neutrons.

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Equilibrium of Newtonian Theory

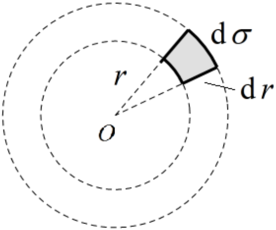


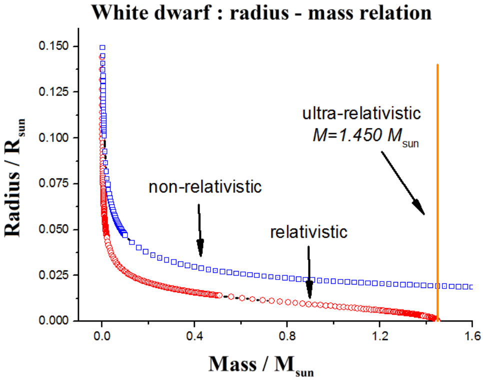
Figure: Sketch of White Dwarf

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP}{dr} = -\frac{G\rho(r)\, m(r)}{r^2}$$

Solution of White Dwarfs

Combine the equilibrium equation and equation of states, we are able to obtain the numerical solution of M-R relation.



Solution of White Dwarfs

The density distribution in the interior of white dwarfs is presented below

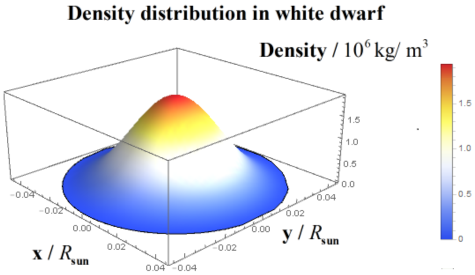


Figure: Density Distribution

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Cooling Equations

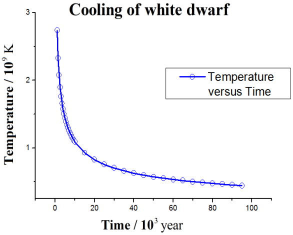

Figure: Cooling Curve

$$T_{\text{critical}} = [\frac{L_\gamma}{2 \times 10^6 M/M_\odot}]^{2/7}(\text{K})$$

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = L_\gamma \Rightarrow -\left(\frac{3k}{2Am_BC}\right)\frac{dT}{dt} = T^{3/2}$$

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Equation of Equilibrium

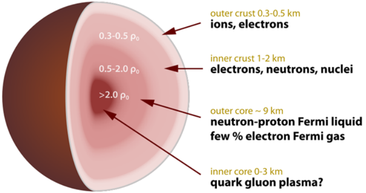

Figure: The possible interior structure of neutron star

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -(\rho(r) + \frac{P(r)}{c^2})\frac{G[m(r) + \frac{4\pi r^3 P(r)}{c^2}]}{r(r - \frac{2Gm(r)}{c^2})}$$

Equation of States: Neutron Stars

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Solution Corresponding to the Oppenheimer's EOS

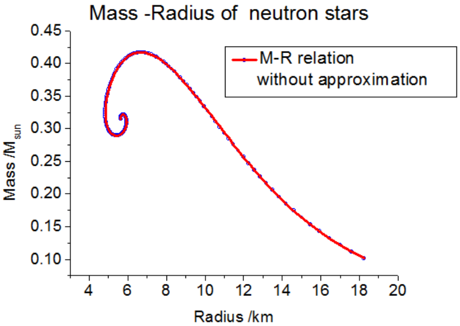

Figure: Numerical Solution of TOV Equation

Solution Corresponding to the Approximation of Oppenheimer's EOS

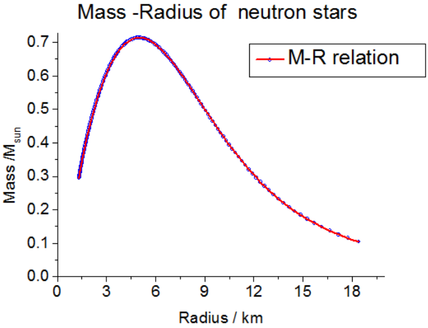

Figure: Numerical Solution of TOV Equation

A Qualitatively Illustration of the Increase of Maximum Mass

Equilibrium: Gravity against degeneracy pressure.
Neutron's degeneracy pressure in the two cases are given by:

$$P = K\rho^{4/3} \quad \text{Relativistic Case}$$

$$P = K\rho^{5/3} \quad \text{Non-Relativistic Case}$$

Relativistic effects result in a weaker degeneracy pressure at a given density.

Oppenheimer's Original Results

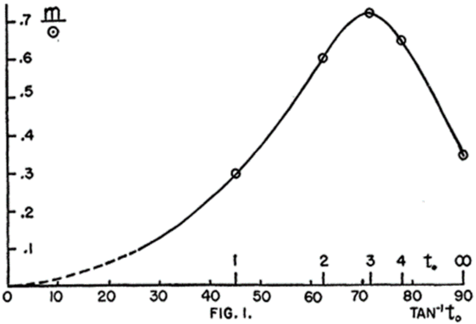

Figure: The solution of Oppenheimer and Volkoff.

Cooling of Neutron Stars

In general, the cooling equation is:

$$\alpha T \frac{dT}{dt} = -bT^k - cT^2$$

The two terms in the equation correspond to neutrino emission and photon emission respectively.

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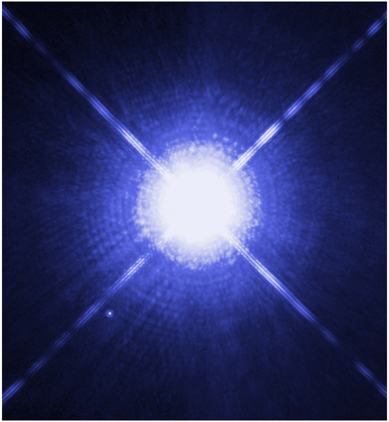

Figure: Hubble Telescope's Photo of Sirius A and B

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Figure: Hubble Telescope's Photo of NGC 2440: Pearl of a New White Dwarf

White Dwarfs on H-R Diagram

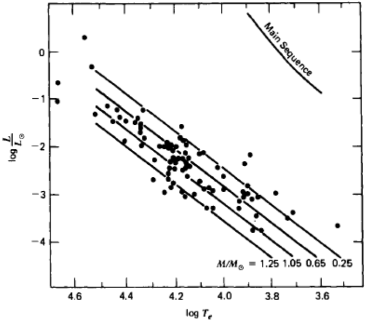

Figure: White Dwarfs on H-R Diagram

According to *Stefan-Boltzmann Law*, we should have:

$$\log L \propto 4 \log T_e$$

Therefore, white dwarfs are expected to occupy a narrow strap in *H-R diagram*.

Measuring the Radii of White Dwarfs

According to energy conservation in radiation, we have:

$$F_\nu(\text{Measured}) = F_\nu(\text{Surface})\frac{R^2}{D^2}$$

In this formula, all quantities except R can be directly or indirectly measured by observations.

Measuring the Masses of White Dwarfs

Using Kepler's 3rd Law

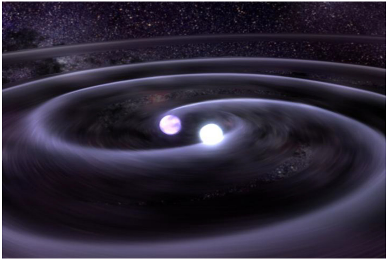

Figure: White Dwarfs Binary System

Invoking:

$$P = 2\pi(\frac{G(M_1 + M_2)}{a^3})^{-1/2}$$

$$M_1 a_1 = M_2 a_2$$

Measuring the Masses of White Dwarfs

Using Gravitational Redshift

The formula is given below:

$$\frac{\Delta\lambda}{\lambda} = \frac{GM}{Rc^2}$$

Remark

In order to distinguish redshift effects from Doppler effects, physicists usually use white dwarfs in wide binaries or common proper-motion pairs, for these white dwarfs' velocities can be measured accurately.

Mass–Radius Relation as Experimental Result

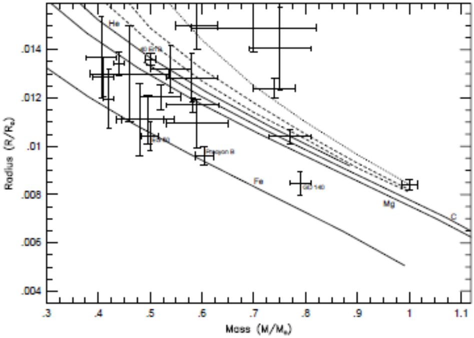

Figure: Mass–Radii Relation[1]

[1] cited from Shipman's paper

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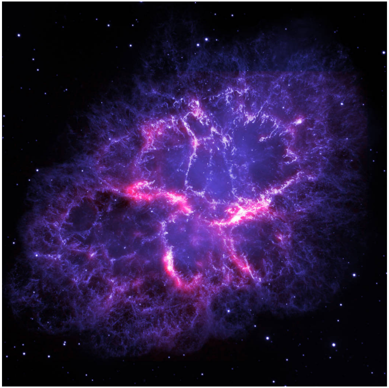

Figure: Crab Nebula: A Supernova Remnant

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Figure: Vela Pulsar: A Pulsar in a Supernova Remnant

Measuring Neutron Stars' Masses

Using Kepler's 3rd Law

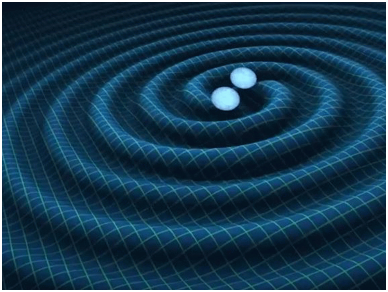

Figure: Neutron Stars Binary System

Invoking:

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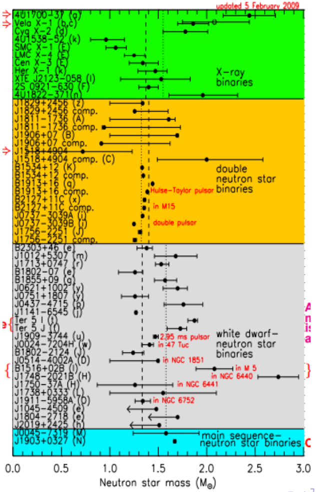
$$M_1 a_1 = M_2 a_2$$

Neutron Stars' Masses as Experimental Results



- Lorimer, D. R., Kramer, M., *Handbook of Pulsar Astronomy*, (London: Cambridge University Press, 2005).
- Chau, W. Y., *Ap. J., 147, 664*, (1967).
- Woltjer, L., *Ap. J., 140, 1309*, (1964).
- Lorimer, D. R., et.al, *Mon. Not. R. Astron. Soc. 372, 777–800*, (2006).
- Lorimer, D. R., *Young Neutron Stars and Their Environments IAU Symposium, Vol. 218*, (2004).
- Stuart L. Shapiro and Saul A. Teukolsky, *Black holes, white dwarfs, and neutron stars: the physics of compact objects*, (New York: Wiley, 1983).

- Provencal, J. L., Shipman, H. L., et.al, *The Astrophysical Journal, 494 :759–767*, (1998).
- Kippenhahn, J., Weigert, A., Weiss, A., *Stellar Structure and Evolution*, (Berlin: Springer, 2012).
- Sedrakian, A., *Lecture on Astroparticle Physics*, (2015).
- Wald, R., *General Relativity*, (Chicago: the University of Chicago Press, 1984).
- Carrol, S., *Spacetime and Geometry*, (Addison-Wesley, 2004).
- Lattimer, J.M., *Neutron Star Equations of State*, (Addison-Wesley, 2009).

Oppenheimer, R., Volkoff, G.M., *Physical Review.55.374-381*, (1939).

Silbar, R., Reddy, S., *American journal of physics.72.892*, (2004).